KWAZULU-NATAL PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

CURRICULUM GRADE 10 -12 DIRECTORATE

NCS (CAPS) SUPPORT

JUST IN TIME LEARNER REVISION DOCUMENT

MATHEMATICS

GRADE 11

2024

This document has been compiled by the FET Mathematics Subject Advisors together with Top Teachers.

TABLE OF CONTENTS

	TOPICS	PAGE NUMBERS
1.	ALGEBRA	3 - 9
2.	NUMBER PATTERNS	10 - 13
3.	FUNCTIONS	14 - 23
4.	FINANCE GROWTH AND DECAY	23 - 26
5.	PROBABILITY	27 - 32
6.	STATISTICS	33 - 50
7.	ANALYTICAL GEOMETRY	41 - 51
8.	TRIGONOMETRY	52 - 64
9.	EUCLIDEAN GEOMETRY	64 - 76
10.	MEASUREMENT	77 - 78

TOPIC	1. ALGEBRA

GUIDELINES, SUMMARY NOTES, & STRATEGIES

Rational exponents (fractional exponents): expressions with exponents that are rational numbers

➢ **Laws of exponents**

- $a^n \times a^m = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $\left(a^m\right)^n = a^{mn}$
- $(a \times b)^n = a^n \times b^n$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^0 = 1$
- $x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$
- **NOTE:** when adding or subtracting powers with numerical bases, **factorise.**
- when $a^m = a^n \therefore m = n$ and $a^m = b^m \therefore a = b$
- when $x : x^{\frac{p}{q}} = t^r \therefore x : x^{\frac{p}{q} \times \frac{q}{p}} = t^{r \times \frac{q}{p}}$ provided p is not an even number

➢ **Quadratic exponents**

It is an equation with three terms, one term has the highest power that is multiple of 2, the middle term has one- half the exponent of the term with a highest power

$x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 3 = 0$

let $x^{\frac{1}{3}} = k$

$k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

➢ **Adding, subtracting, dividing and multiplying surds**

- $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- $p\sqrt[n]{a} \pm q\sqrt[n]{a} = p \pm q\sqrt[n]{a}$

➢ **Factorisation:**

- common factor
- trinomial
- difference of two squares
- grouping
- sum and difference of two cubes

➢ **Solving surd equation**

Steps:

- isolate the surd
- square both sides
- simplify and solve the equation
- reject if applicable

➢ **Completing the square**
steps

- "take out" the coefficient of x^2 for the first 2 terms $a\left(x^2+\frac{b}{a}\right)x+c$
- Add and subtract $\left(\frac{1}{2}\times(\textit{the coefficient of } x)\right)^2$ $a\left(x^2+\frac{b}{2a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right)+c$
- Factorise the perfect square trinomial and multiply $a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+c$

➢ **Quadratic equation**

from completing the square

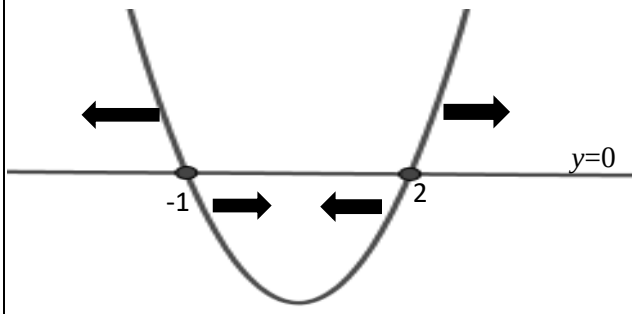
$$a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+c=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}=-\frac{c}{a}$$

If $ax^2+bx+c=0;\ \therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a\neq 0$

➢ **Inequalities (notes)**



- Where $y\geq 0$, above x-axis: $x\leq -1$ or $x\geq 2$
- Where $y\leq 0$, below the x-axis: $-1\leq x\leq 2$

Simultaneous equations	
When solving a pair of simultaneous linear equations, we are, in fact, finding a common point – the point of intersection of the two lines.	
Elimination method	**Substitution method**
• Make the coefficients of one of the variables the same in both equations, Eliminate the variable by adding equation (1) and equation (2) together. • Simplify and solve for x. • Substitute x back into either original equation and solve for y.	• Use the simplest of the two given equations to express one of the variables in terms of the other. • Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one. • We now have one equation with one unknown variable which can be solved. • Use the solution to substitute back into the first equation to find the value of the other unknown variable.

Nature of roots

$ax^2 + bx + c = 0 \therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- $b^2 - 4ac = 0$ equal roots
- $b^2 - 4ac < 0$ non real
- $b^2 - 4ac \geq 0$ real and equal
- $b^2 - 4ac =$ perfect square rational roots
- $b^2 - 4ac \neq$ perfect square irrational roots

ACTIVITIES

1.1 **Simplify**

1.1.1 $\dfrac{4^{x+2}.8^{x-1}}{2^{x-3}.16^{x}}$ (4) **L2**

1.1.2. $\dfrac{15^{n+2}.45^{1-n}}{3^{3-n}}$ (4) **L2**

1.1.3 $\dfrac{2^{x} + 2^{x+1}}{2^{x} - 2^{x+2}}$ (5) **L2**

1.1.4 $\dfrac{25^{x+1}.6}{10^{x-1}.15^{x}}$ (4) **L2**

1.1.5 $\dfrac{6^{n+3}.2^{n-1}}{12^{n+2}}$ (3) **L2**

1.1.6 $\dfrac{2^{4x+1}.9^{x}.6^{2x-1}}{12^{3x}.3^{x}}$ (4) **L2**

1.2 **Convert the following into exponential form**

1.2.1 a. $\sqrt{x}$ (1) **L1**

b. $\sqrt[3]{x^2}$ (1) **L1**

1.3 **Solve for x**

1.3.1 $x^{\frac{3}{4}} = 8$ (3) **L1**

1.3.2 $3^{x+2} + 3^{x+1} = 1$ (4) **L2**

1.3.3 $x^{\frac{1}{2}} - 3x^{\frac{1}{4}} - 10 = 0$ (4) **L2**

1.3.4 $x^{-\frac{2}{3}} = 0,25$ (3) **L2**

1.3.5 $x^{\frac{2}{3}} = 4$ (2) **L1**

1.3.6 $x^{\frac{1}{3}} = -1$ (2) **L1**

1.3.7 $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} - 3 = 0$ (4) **L3**

1.3.8 $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 4 = 0$ (4) **L3**

1.3.9 $3x^{\frac{3}{4}} = 24x^{\frac{1}{4}}$ (4) **L2**

1.3.10 $x - 4x^{\frac{1}{2}} = 5$ (5) **L3**

1.3.11 $2^{x-2} + 2^x + 2^{x+1} - 52 = 0$ (4) **L2**

1.4 **Simplification**

1.4.1 $\sqrt{18} + \sqrt{50} - \sqrt{8}$ (3) **L1**

1.4.2 $\dfrac{\sqrt{132}}{\sqrt{3}}$ (3) **L1**

1.4.3 $\left(\sqrt{5+3}\right)\left(\sqrt{5-3}\right)$ (3) **L2**

1.4.4 $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$ (3) **L2**

1.4.5 $\dfrac{\sqrt{48}-\sqrt{3}}{\sqrt{27}}$ (2) **L1**

1.4.6 $(2+3\sqrt{32})^2$ (3) **L2**

1.5 **Solve for x**

1.5.1 $\sqrt{x-3} - 4 = 0$ (3) **L2**

1.5.2 $\sqrt{x+2} = x - 4$ (4) **L2**

1.5.3 $\sqrt{10-x} = -2x - 1$ (4) **L2**

1.5.4 $\dfrac{\sqrt{x}}{\sqrt{2}} = 3\sqrt{2}$ (4) **L2**

1.5.5 $\sqrt{12+x} = \sqrt{x} + 3$ (4) **L3**

1.5.6 $\sqrt{8-x} = 2$ (2) **L2**

1.5.7 $2\sqrt{\dfrac{x}{2} - 3} + 4 = 12$ (5) **L3**

1.5.8 $4\sqrt{x-3} - 3 = 2x - 9$ (5) **L2**

1.5.9 $\sqrt{2x-1}-\frac{3}{\sqrt{2x-1}}=-2$ (4) **L3**

1.5.10 $2\sqrt{4-x}=2\sqrt{4-x}$ (4) **L1**

1.5.11 $3\sqrt{\sqrt{x}+3}=9$ (4) **L3**

1.5.12 $\sqrt{2-7x}+2x=0$ (4) **L2**

1.6 **Solve for *x* through factorisation**

1.6.1 $x^2=2x$ (2) **L1**

1.6.2 $x^2=2x+3$ (3) **L1**

1.6.3 $7x^2+18x=9$ (3) **L2**

1.6.4 $x(x+1)=6$ (4) **L2**

1.6.5 $(4-x)(x-2)=0$ (2) **L1**

1.6.6 $(4+x)(3-x)=-8$ (3) **L2**

1.6.7 $\frac{x^2-1}{x+1}=2$ (4) **L2**

1.7 **Solve for x**

1.7.1 $x^2\le 2x$ (3) **L2**

1.7.2 $x^2\ge 2x+3$ (3) **L2**

1.7.3 $7x^2+18x\le 9$ (4) **L2**

1.7.4 $x(x+1)\ge 6$ (4) **L2**

1.7.5 $(4-x)(x-2)\le 0$ (3) **L2**

1.7.6 $2^x(x-3)\le 0$ (3) **L3**

1.7.7 $2^x(x^2-9)<0$ (3) **L3**

1.7.8 $x^2+2x-20<-x-2$ (4) **L2**

1.7.9 $5<x(2x+3)$ (4) **L2**

1.7.10 $-\frac{x^2}{2}-x>-4$ (4) **L2**

1.8 **Solve for *x* and *y* simultaneously**

1.8.1 $2x^2-3xy=-4$ and $4^{x+y}=2^{y+4}$ (5) **L2**

1.8.2 $x^2-2xy-3y^2=0$ and $y-x=2$ (6) **L2**

1.8.3 $3^{2x}=3^{y-1}$ and $2x+2y=4$ (5) **L2**

1.8.4 $y-4=2x$ and $2x^2-3xy+y^2=4$ (5) **L2**

1.8.5 $(3^x - 9)(\sqrt{y-4}) = 0$ (4) **L2**

1.8.6 $(x-9)^2(y+8)^2 = 0$ (4) **L2**

1.8.7 $2^{x^2} = \frac{32}{2^y}$ and $x - y - 1 = 0$ (7) **L3**

1.8.8 $x - 2y = 2$ and $y = (x+1)(x-3)$ (6) **L2**

1.8.9 $x^2 + 5xy + 6y^2 = 0$ calculate the value of $\frac{x}{y}$ (5) **L3**

1.9 **Solve for *x* and give your answer correct to 2 decimal places**

1.9.1 $x(x+2) - 4 = 2$ (4) **L2**

1.9.2 $-x^2 = 2 - 7x$ (4) **L2**

1.9.3 $x^2 = 2x + 4$ (4) **L2**

1.9.4 $x^2 = 2x + 7$ (4) **L2**

1.9.5 $7x^2 + 18x = 9$ (4) **L2**

1.9.6 $x(x+1) = 3$ (5) **L2**

1.9.7 $(4-x)(x-7) = 59$ (5) **L2**

1.9.8 $\sqrt{2x-1} = \frac{x}{\sqrt{3x+2}}$ (5) **L3**

1.9.9 **Solve for *x* by completing the square [solving an equation by completing the square will not be examined]**

(a) $x^2 = 2x$ (3) **L2**

(b) $x^2 = 2x + 3$ (3) **L2**

(c) $7x^2 + 18x = 9$ (4) **L2**

(d) $x(x+1) = 6$ (4) **L2**

1.10 **Nature of roots**

1.10.1 For which values of *k* will $kx^2 - 6x - 3 = 0$ have non-real roots? (3) **L2**

1.10.2 Given the equation $4x^2 + kx + 1 = 0$ has equal roots, determine the value of *k* (where $k < 0$). (4) **L2**

1.10.3 For which values of *x* will $x = \sqrt{(3-x)(x+5)}$ be real? (4) **L3**

1.10.4 Show that $\sqrt{\frac{a^2 - b^2}{a+b}}$ is non-real for $b > a$. (4) **L3**

1.10.5 Given: $f(x) = ax^2 + bx + c$. (5) **L4**

$f(x) = 0$ has equal roots, namely : $x = \frac{12 \pm \sqrt{192 - 48k}}{2k - 1}$

If it is further given that $f(-1) = 27$, determine the values of a, b and c.

1.10.6 Show that the roots of $2x^2 + (1-2n)x - n = 0$ are rational and unequal if n is an integer. (3) **L4**

1.10.7 $k = \sqrt{(x+1)^2 - 4}$, where k is a real number. Write down the minimum value of k. (1) **L1**

1.10.8 Calculate the values of k, for which the equation with roots (3) **L2**

$x = \frac{-2 \pm \sqrt{4 - 12(-k+1)}}{6}$ will have real roots.

1.11 **Problem solving questions**

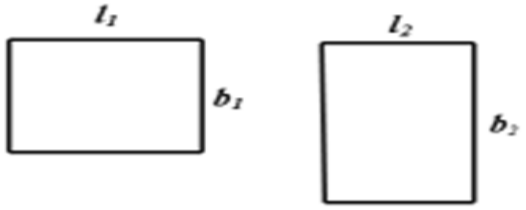
1.11.1 The roots of $y = \frac{2}{y-1}$ are $y = 2$ and $y = -1$. Hence, or otherwise, determine the roots of $x^2 + 2x - 1 = \frac{2}{x^2 + 2x - 2}$. (5) **L3**

1.11.2 If $3^{9x} = 64$ and $5^{\sqrt{p}} = 64$, WITHOUT the use of a calculator, calculate the value of $\frac{\left[3^{x-1}\right]^3}{\sqrt{5}^{\sqrt{p}}}$ **L4**

1.11.3 Given the expression: $\frac{8}{1 - \frac{1}{\sqrt{2}}} = a + b\sqrt{c}$ (5) **L4**

Show, without the use of a calculator, that $a = bc$

1.11.4 The two rectangles drawn below have the same area. The first rectangle has a breadth that is 1cm more than its length. The second rectangle has a breadth that is 4cm more than its length. The length of the second rectangle is 1cm less than the length of the first rectangle. Determine the areas of the rectangles. (4) **L4**



1.11.5 If $x = \sqrt{3 - 2\sqrt{2}}$ and $y = \sqrt{2} - 1$, show that $x = y$. (5) **L3**

1.11.6 Calculate a and b if $\sqrt{\frac{5^{2014} - 5^{2012}}{6}} = a(5^b)$ and a is not a multiple of 5. (4) **L3**

1.11.7 Given $a - b = 3$ and $3^a + 9.3^b = 4$, solve simultaneously for a and b. (6) **L3**

1.11.8 *if* $4^x = 8^y$ find the ratio of $x:y$ (3) **L3**

TOPIC	2. NUMBER PATTERNS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

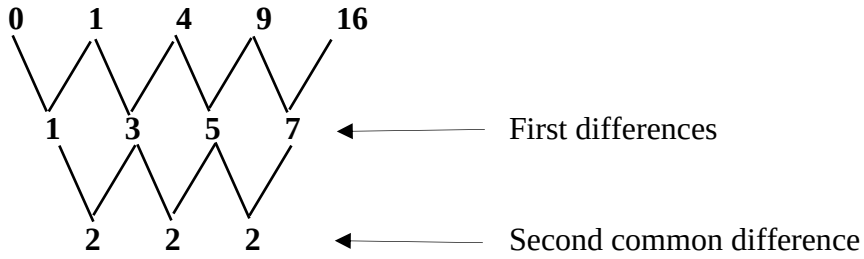
A linear pattern is a pattern where the difference (d) between consecutive terms is common.

$d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = ...$

The general term of a linear pattern can be given by $T_n = dn + c$ where d is the common difference.

A quadratic pattern is a pattern with a common second difference. The **first differences** of a quadratic pattern form a **linear pattern.**

Example:



1. Determining the next term.

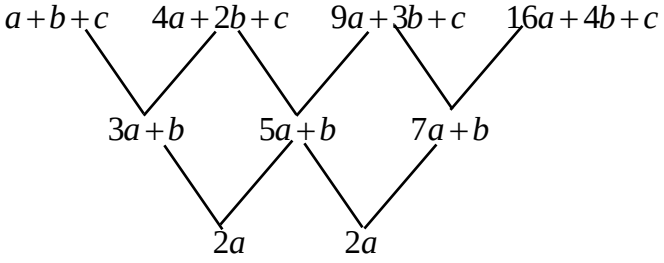
To determine the next term, we add the last terms of each of the sequences, i.e. the last of the quadratic pattern to the last term of the first differences and the second common difference.

The next term is $16 + 7 + 2 = 25$

2. Determining the n^{th} term.

The n^{th} term of the quadratic pattern is given by: $T_n = an^2 + bn + c$

Using this formula to generate the terms we will have:



- $2a =$ the second common difference
- $3a + b =$ the first term of the first differences
- $a + b + c =$ the first term of the quadratic pattern

From the example above:

$2a = 2$	$3a + b = 1$	$a + b + c = 0$
$a = 1$	$3(1) + b = 1$	$1 - 2 + c = 0$
	$b = -2$	c = 1

$\therefore T_n = n^2 - 2n + 1$

4. Determining T_n and n.

It is important to note that $n \in \mathrm{N}$, that means it can **NEVER** be negative, irrational or a fraction.

(a) Determining the 15^{th} term of the sequence

$T_{15} = (15)^2 - 2(15) + 1$

$T_{15} = 196$

(b) Determining which term of the sequence is 81.

$81 = n^2 - 2n + 1$

$n^2 - 2n - 80 = 0$

$(n-10)(n+8) = 0$

$n = 10$ or $n = -8$

$n \neq -8$

$\therefore n = 10$

ACTIVITIES

2.1 Given the linear pattern: $5\ ;\ 9;\ 13\ldots$

2.1.1 Write down the next two terms. (2) **L1**

2.1.2 Determine the n^{th} term. (3) **L2**

2.1.3 Calculate the value of the 25^{th} term. (2) **L1**

2.1.4 Which term in the sequence has a value of 241? (3) **L2**

2.2 Consider the linear pattern: $55\ ;\ 48\ ;\ 41...$

2.2.1 Write down the common difference. (1) **L1**

2.2.2 Determine the value of n if $T_n = -169$ (4) **L2**

2.3 Consider the linear pattern: $102\ ;\ 97\ ;\ 92\ ;.....-133$

2.3.1 Determine the general term of the pattern. (3) **L2**

2.3.2 How many terms are there in the pattern? (3) **L2**

2.4 Consider the linear sequence: $11\ ;\ x\ ;\ 23\ldots$

2.4.1 Calculate the value of x. (2) **L2**

2.4.2 Write down the general term. (3) **L2**

2.4.3 Determine T_{15} (2) **L1**

2.5 The first three terms of a linear sequence are $5p-8\ ;\ 3p+3\ ;\ 4p+2$

2.5.1 Determine the value of p. (3) **L2**

2.5.2 Determine the values of the first three terms. (1) **L1**

2.5.3 Determine a formula for the n^{th} term. (3) **L2**

2.6 Consider the linear pattern: $x\ ;\ 29\ ;\ y\ ;\ 41$

Calculate the values of x and y. (4) **L2**

2.7 The second term of a linear pattern is 9 and the 19^{th} term is 43.

Calculate the value of the first term and the common difference. (3) **L3**

2.8 Given that the 5^{th} term of a linear pattern is 7 and the sum of the 11^{th} and 17^{th} terms is 68.

2.8.1 Calculate the value of the first term and the common difference. (5) **L3**

2.8.2 Determine T_{25}. (2) **L1**

2.9 Given a quadratic pattern: $22\ ;\ 1\ ;\ -18\ ;\ -35$

2.9.1 Write down the next 2 terms. (2) **L1**

2.9.2 Determine the formula for the n^{th} term. (4) **L2**

2.9.3 Which term(s) will have the value of -95 ? (4) **L2**

2.9.4 Which term will have the smallest value? (2) **L3**

2.10 Given a quadratic pattern: $121\ ;\ -83\ ;\ -49\ ;\ -19...$

2.10.1 Write down the next two terms. (2) **L1**

2.10.2 Determine the formula for the n^{th} term. (4) **L2**

2.10.3 Which term has a value of -13369? (4) **L2**

2.10.4 Between which two consecutive terms is the first difference -110? (3) **L3**

2.10.5 Determine the maximum value of the pattern. (4) **L3**

2.11 Consider the quadratic pattern: $5\ ;\ 6\ ;\ 11\ ;\ 20.....$

2.11.1 Write down the next two terms. (2) **L1**

2.11.2 Determine the formula for the n^{th} term. (4) **L2**

2.11.3 Is 2321 a term in the pattern? (4) **L3**

2.11.4 Between which two consecutive terms will the first difference be 69? (3) **L3**

2.12 The first four terms of a quadratic pattern are: $10\ ;\ 10\ ;\ 12\ ;\ 16$

2.12.1 Determine the value of the 29^{th} term. (5) **L2**

2.12.2 Determine the general term for the first differences. (2) **L2**

2.13 $-10\ ;\ -25\ ;\ 38\ ;-49....$ is a quadratic pattern.

2.13.1 Determine the value of T_{18}. (5) **L2**

2.13.2 Which term(s) has a value of -70? (4) **L2**

2.13.3 Which term is first to be positive? (4) **L3**

2.14 $2\ ;\ 6\ ;\ 10$ ..... are the first differences of a quadratic pattern

2.14.1 Determine the first difference between T_{37} and T_{38} of the quadratic pattern. (3) **L2**

2.14.2 If the 19^{th} term of a quadratic pattern is 651, prove that the general term will be given by $T_n = 2n^2 - 4n + 5$ (4) **L3**

2.14.3 Prove that all terms of the quadratic pattern will be odd. (2) **L4**

2.15 The general term of the first differences of a quadratic pattern is given by $T_k = -4k + 29$

2.15.1 Between which two consecutive terms of the quadratic pattern will the first difference be -71? (2) **L2**

2.15.2 If the first term of the quadratic pattern is 15, determine the general term of the quadratic pattern. (4) **L3**

2.16 Consider the quadratic pattern: $x\ ;\ 4\ ;\ y\ ;\ 26\ ;\ 46....$

2.16.1 Calculate the value(s) of x and y. (5) **L3**

2.16.2 If $x = 2$ and $y = 12$, determine the n^{th} term of the pattern. (4) **L2**

2.16.3 Will there be negative terms in the pattern? (3) **L3**

2.17 The first term of a quadratic pattern is 10, the third and fourth terms are -6 and -11 respectively.

2.17.1 Determine the value of the second and fifth term. (4) **L3**

2.17.2 Determine the n^{th} term. (4) **L2**

2.18 The first four terms of a quadratic pattern are: $23\ ;\ x\ ;\ y\ ;\ 65$ and the second difference is 2.
Show that $x = 35$ and $y = 49$. (5) **L3**

2.19 $3x+1\ ;\ 2x\ ;\ 3x-7$ are the 11^{th}, 12^{th} and 13^{th} terms of the linear pattern.

2.19.1 Determine the value of x. (3) **L2**

2.19.2 Calculate the numerical values of the first three terms. (3) **L1**

2.20 The following information is given about a quadratic pattern:

$T_5 = 121$

$T_6 - T_5 = 37$

$T_7 - T_6 = 41$

2.20.1 Write down the first term of the quadratic pattern. (2) **L2**

2.20.2 Determine an expression of the general term. (4) **L2**

2.21 The first four terms of a quadratic pattern are: $2x-4\ ;\ x+2\ ;\ 3x+1\ ;\ 6x-1$

2.21.1 Calculate the value of x. (5) **L3**

2.21.2 If $x = 3$, write down the numerical values of the first three terms. (1) **L1**

2.22.3 Hence, determine the n^{th} term of the pattern. (4) **L2**

2.22 $4x+3\ ;\ 2x-5\ ;\ -2x-11\ ;\ x+3$ are the first four terms of a quadratic pattern

2.22.1 Show that $x = -2$ (5) **L2**

2.22.2 Determine an expression for the general term. (4) **L2**

2.22.3 Determine T_{35} (2) **L1**

2.23 A pattern with a constant second difference has $T_n = 4n+3$ as the general term of its first differences. The first term of the quadratic pattern is 7.
Determine the general term of the quadratic pattern. (5) **L3**

2.24 A quadratic pattern has a second difference of 2, $T_4 = -25$ and $T_{11} = -158$
Determine the formula for the n^{th} term. (6) **L4**

TOPIC	**3. FUNCTIONS AND GRAPHS**		
GUIDELINES, SUMMARY NOTES, & STRATEGIES			
Straight Line	**Parabola**	**Hyperbola**	**Exponential**
$y = mx + c$ m … **gradien**t and c … ***y*-intercept**	$y = a(x+p)^2 + q$ **Axis of symmetry with equation** $x = -p$ **Maximum** or **minimum value** $(-p;q)$ Turning point	$y = \frac{a}{x+p} + q$ **Vertical asymptote:** **Horizontal asymptote:** $y = q$	$y = a.b^{x+p} + q$ $b > 0$ **and** $b \neq 1$ **Horizontal asymptote with equation** $y = q$
$m < 0$ **… graph is decreasing** $m > 0$ **… graph is increasing** **IF *m* = 0** **THEN** $y = c$ **IF** m is undefined **THEN *x* = *c***	$a < 0$ **… graph faces downwards(concave down) and has a minimum turning point** $a > 0$ **… graph faces upwards(concave up) and has a maximum turning point**	$a < 0$ **… graph is on the second and the fourth quadrant** $a > 0$ **… graph is on the first and the third quadrant**	$a < 0$ **… graph is below the asymptote** $a > 0$ **… graph is above the asymptote**
Domain: $x \in R$ **Range:** $y \in R$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$	**Domain:** $x \in R$, $x \neq -p$ **Range:** $y \in R$, $y \neq q$	**Domain:** $x \in R$ **Range:** $y > q$ **if** $a > 0$ $y < q$ **if** $a < 0$
$y - y_1 = m(x - x_1)$	$y = ax^2 + bx + c$ **Axis of symmetry:** $x_M = \frac{-b}{2a}$ **x-intercepts** $y = a(x - x_1)(x - x_2)$ x_1 **and** x_2 **are**	**Axis of symmetry/lines of symmetry:** $\left\{ \begin{matrix} y = x + c \\ y = -x + c \end{matrix} \right\}$ **OR** $y = \pm(x - p) + q$	

- ✓ Understand reflection about the *y*-axis: $g(x) = f(-x)$ and about the *x*-axis $g(x) = -f(x)$
- ✓ Understand reflection about a line and about a point (use midpoint formula)
- ✓ Understand vertical translation, $h(x) = f(x) + q$, and horizontal translation $h(x) = f(x + p)$
- ✓ If given a sketch of a function $f(x)$, be able to determine the values of x for which:
  $f(x) < 0$ OR $f(x) > 0$
- ✓ If given the graphs of two functions, $f(x)$ and $g(x)$, on the same set of axes (intersecting graphs), be able to determine the values of x for which:
  1. $f(x) = g(x)$
  2. $f(x) < g(x)$ or $f(x) - g(x) < 0$
  3. $f(x) > g(x)$ OR $f(x) - g(x) > 0$
  4. $f(x).g(x) > 0$ or $f(x).g(x) < 0$
  5. $\frac{f(x)}{g(x)} > 0$

ACTIVITIES

QUESTION 3

3.1

Given: $h(x)=\dfrac{-3}{3-x}-2$

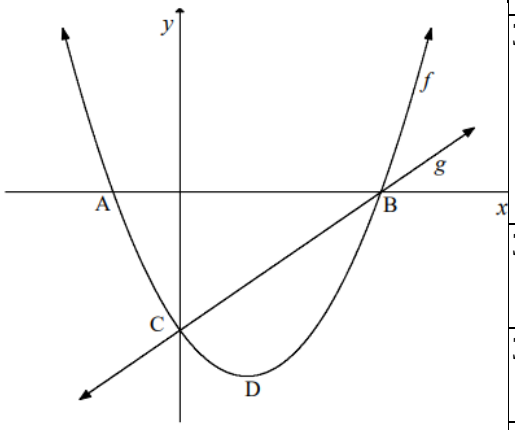
3.1.1	Write down the equations of the asymptotes of *h*.	(2)	**L2**
3.1.2	Write down the coordinates of the *y*-intercept of *h*.	(2)	**L1**
3.1.3	Calculate the coordinates of the *x*-intercepts of *h*.	(3)	**L2**
3.1.4	Draw the graph of *h*. Clearly show the asymptotes and the intercepts with the axes.	(3)	**L2**
3.1.5	Determine the equation of the axis of symmetry of *h* that has a positive gradient.	(2)	**L2**
3.1.6	Calculate the *x*-coordinates of the points of intersection of *h* and the axis of symmetry determined in QUESTION 3.1.5.	(5)	**L3**
		[17]	

3.2 **GRADE 11 KZN NOV 2023**

The sketch below shows the graphs of:

$f(x)=\dfrac{1}{2}(x-2)^2-8$ and $g(x)=x-6$.

- A and B are *x*-intercepts of *f*.
- D is a turning point of *f*.
- C is the *y*-intercept of both graphs.
- Graphs *f* and *g* intersect at B and C.



3.2.1	Calculate the length of AB.	(4)	**L2**
3.2.2	Write down the coordinates of D.	(1)	**L1**
3.2.3	Write down the range of *f*.	(1)	**L1**
3.2.4	Calculate the average gradient between points A and C.	(3)	**L2**
3.2.5	For which values of k will $\dfrac{1}{2}x^2-2x-6-k=0$ have two positive roots?	(2)	**L3**
3.2.6	Use the graphs to determine the values of x for which:		
	(a) $f(x)\geq 0$	(2)	**L2**
	(b) $\dfrac{g(x)}{f(x)}>0$	(2)	**L3**
3.2.7	Determine the maximum value of h if $h(x)=\sqrt{2}^{2-f(x)}$.	(3)	**L3**
3.2.8	Calculate the value(s) of k for which $y=2x+k$ will NOT be a tangent of f.	(5)	**L4**
		[23]	

3.3 **DBE NOV 2016**

Given: $f(x)=-2x^2+x+6$

3.3.1	Determine *y*-intercept of *f*.	(1)	**L1**
3.3.2	Determine the *x*-intercept of *f*.	(3)	**L2**
3.3.3	Determine the coordinates of the turning point of *f*.	(3)	**L2**
3.3.4	Sketch the graph of *f* showing all the intercepts with the axes and the turning point.	(4)	**L2**
3.3.5	Determine the value(s) of k for which $f(x)=k$ will have equal roots.	(2)	**L2**
3.3.6	If the graph of *f* is shifted 2 units to the right and 1 unit upwards to form *h*, determine the equation of *h* in the form of $y=a(x+p)^2+q$.	(3)	**L3**
		[16]	

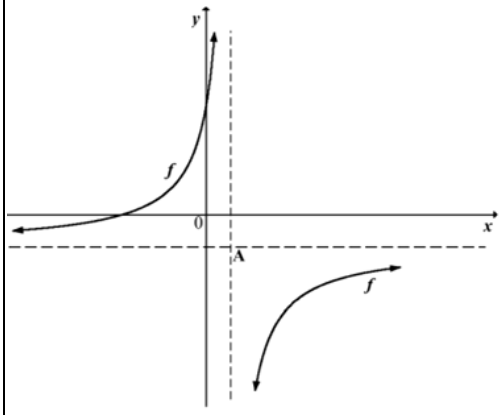
3.4 **PHOENIX CENTRAL CLUSTER 2022**

Consider the following functions:
$f(x) = 3^{x+1} - 1$ and $g(x) = -2x + 2$

3.4.1	Write down the equation of the horizontal asymptote of *f*.	(1)	**L1**
3.4.2	Determine the coordinates of x and y intercepts of *f*.	(4)	**L2**
3.4.3	Sketch the graphs of *f* and *g* on the same set of axes.	(5)	**L2**
3.4.4	Determine the value(s) of x for which $f(x) - g(x) \leq 0$.	(2)	**L2**
3.4.5	If the graph of *f* is reflected about the *x*-axis and shifted 3 units upwards to form *h*(*x*), determine the range of *h*.	(3)	**L3**
		[15]	

3.5

The sketch below shows the graph of $f(x) = \frac{-9}{x-1} - 2$. A is the point of intersection if the asymptotes of *f*.



3.5.1	Write down the coordinates of A.		(2)	**L1**
3.5.2	Determine the coordinates of the *x* and *y* intercepts of *f*.		(5)	**L2**
3.5.3	Write down the equation of the axis of symmetry of *f* that has a negative gradient.		(2)	**L2**
3.5.4	Hence, or otherwise, determine the coordinates of a point that lies on *f* in the fourth quadrant, which is the closest to point A.		(5)	**L4**
3.5.5	Determine the value(s) of x for which $f(x) \leq 0$		(2)	**L2**
3.5.6	The graph of *g* is obtained by reflecting the graph of *f* about the *x*-axis.			
	(a)	Write down the equation of *g* in the form $y = \ldots$	(2)	**L1**
	(b)	If A′ is the point of intersection of *g*, write down the coordinates of the point on *g* that is closest A′.		
			[18]	

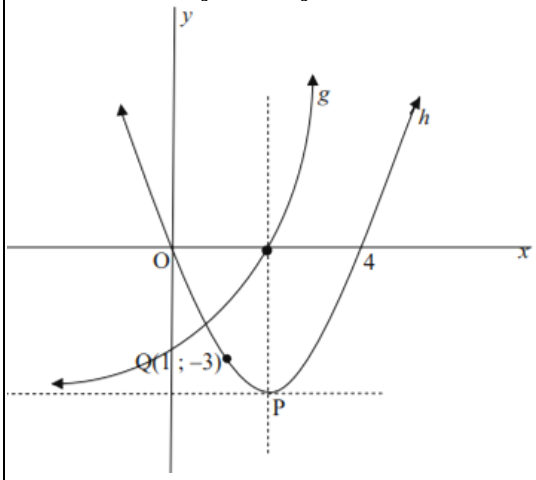
3.6

Given $f(x) = (x+2)^2 - 25$ and $g(x) = -3x - 21$.

3.6.1	Determine *x* and *y* intercepts of *f*.		(4)	**L2**
3.6.2	Determine *x* and *y* intercepts of g.		(2)	**L1**
3.6.3	Determine the coordinates of the turning point of *f*.		(2)	**L1**
3.6.4	Sketch the graphs of *f* and *g* on the same system of axes. Clearly indicate all the intercepts with the axis and turning point.		(5)	**L2**
3.6.5	For which value(s) of x is:			
	(a)	$g(x) = f(x)$?	(1)	**L2**
	(b)	$f(x).g(x) \leq 0$?	(2)	**L2**
3.6.6	Determine the value(s) of k for which $f(x) = g(x) + k$ will have no real roots.		(1)	**L3**
			[17]	

3.7 **KZN JUNE 2019**

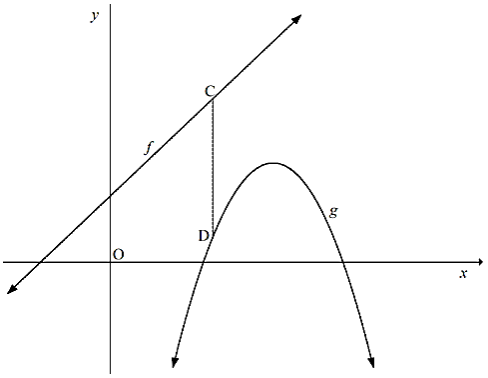
The sketch below shows the graphs of $g(x) = k^x + t$ and $h(x) = ax^2 + bx + c$.
The *x*-intercepts of *h* are (0; 0) and (0; 4)
Q(1 ; –3) is a point on *h*. The horizontal line P, the turning point of *h*, is the asymptote of *g*. The *x*-intercepts of *g* lies on the axis of symmetry of *h*.



3.7.1	Show that $a = 1$ and $b = -4$	(3)	**L3**
3.7.2	Determine the coordinates of P, the turning point of *h*.	(2)	**L2**
3.7.3	write down the range of *h*.	(1)	**L1**
3.7.4	Write down the value of *t*.	(1)	**L1**
3.7.5	Hence, calculate the value of *k*.	(2)	**L2**
3.7.6	For which value(s) of *r* will the roots of $x^2 - 4x + r$ be non-real?	(2)	**L2**
3.7.7	If *h* is reflected in the *x*-axis and shifted 2 units upwards, determine the coordinates of P'.	(2)	**L2**
3.7.8	If *g* is shifted 4 units upwards to form *f* (*x*).		
(a)	Determine the range of *f*.	(1)	**L1**
(b)	Determine the domain of *f*.	(1)	**L1**
		[15]	

3.8 **DBE NOV 2015**

The sketch below shows the graphs of $f(x) = 2x + 3$ and $g(x) = -2x^2 + 14x + k$.
C is any point on *f* and D is any point on *g*, such that CD is parallel to the *y*-axis.
k is a value such that C lies above D.



3.8.1	Determine the equation of the axis of symmetry of g.	(2)	**L2**
3.8.2	Determine the value(s) of *x* for which $f(x) \leq 0$.	(1)	**L2**
3.8.3	Write down a simplified expression for the length of CD in terms of *x* and *k*.	(3)	**L2**
3.8.4	If it is given that the minimum length of CD is 5 units, calculate the value of *k*.	(4)	**L3**
3.8.5	If it is given that $k = -20$		
(a)	Determine the *x*-intercepts of *g*.	(3)	**L2**
(b)	Determine the coordinates of the turning point of *g*.	(3)	**L2**
(c)	Calculate the value(s) of *x* for which $f(x) < 2$.	(4)	**L3**
(d)	Write down the domain and range of *g*.	(2)	**L1**
		[22]	

3.9 **GRADE 12 KZN SEPT 2023**

Given: $g(x) = \dfrac{-3}{3-x} - 1$.

3.9.1	Write down the equations of the asymptotes of *g*.	(2)	**L1**
3.9.2	Calculate the *x* and *y* intercepts of *g*.	(3)	**L2**
3.9.3	Draw the graph of *g*, clearly show all the asymptotes and the intercepts with axis.	(3)	**L2**
3.9.4	Determine the equation of the line of symmetry with a negative gradient.	(2)	**L2**
		[10]	

3.10 **GRADE 12 DBE NOV 2018**

The function f, defined by $f(x)=\frac{a}{x+p}+q,$ has the following properties:

The range of f is $y \in R,\ y \neq 1$.

The graph f passes through the origin.

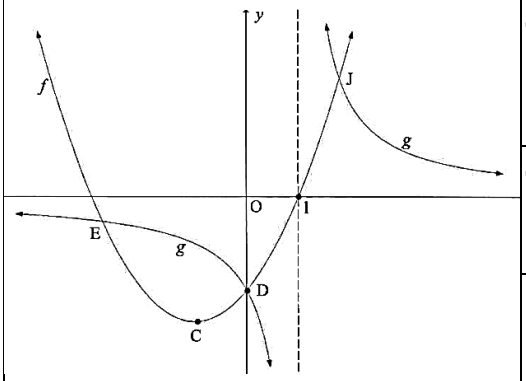
$P(\sqrt{2}+2;\sqrt{2}+1)$ lies on the graph of f.

3.10.1	Write down the values of q.	(1)	**L1**
3.10.2	Calculate the values of a and p.	(5)	**L3**
3.10.3	Sketch a neat graph of this function. Your graph must include the asymptotes, if any.	(4)	**L2**
3.10.4	Determine the equation of the line of symmetry with a positive y-intercept.	(2)	**L2**
3.10.5	If point $P(\sqrt{2}+2;\sqrt{2}+1)$ is reflected about the point of intersection of the asymptotes of f, calculate the coordinates of P'.	(2) (3)	**L2** **L3**
		[15]	

3.11 **GRADE 12 DBE NOV 2019**

Below are the graphs of $f(x)=ax^2+bx-3$ and $g(x)=\frac{a}{x+p}$.

f has a turning point at C and passes through the x-axis at (1; 0). D is the y-intercept of both f and g also intersect each other at E and J. The vertical asymptote of g passes through the x-intercept of f.



3.11.1	Write down the value of p.		(1)	**L1**
3.11.2	Show that $a = 3$ and $b = 2$.		(3)	**L3**
3.11.3	Calculate the coordinate of C.		(4)	**L2**
3.11.4	Write down the range of f.		(2)	**L1**
3.11.5	Determine the equation of the line through C that makes an angle of 45° with the positive x-axis. Write your answer in the form $y =$.....		(3)	**L3**
3.11.6	Determine the value(s) of k for which $f(x)+k$ will be positive.		(2)	**L4**
3.11.7	Determine the value(s) of x for which:			
	(a)	$x.f(x)<0$	(3)	**L2**
	(b)	$g(x)\leq -3$	(2)	**L2**
3.11.8	The function $h(x) = f(m-x)+q$ has only one x-intercept at $x = 0$. Determine the values of m and q.		(4)	**L4**
			[24]	

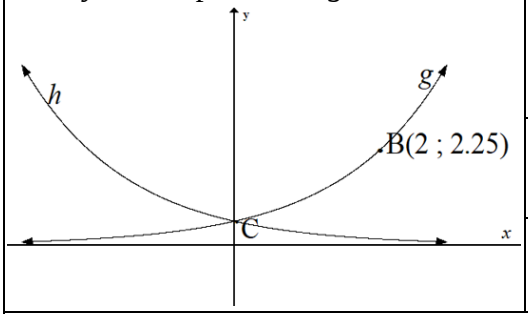
3.12 **GRADE 12 DBE/ MAY-JUNE 2022**

The graph of passes $g(x)=a\left(\frac{1}{3}\right)^x+7$ through point E(– 2; 10).

3.12.1	Calculate the value of a.		(3)	**L2**
3.12.2	Calculate the coordinates of the y-intercept of g.		(2)	**L1**
3.12.3	Consider: $h(x)=(3)^{-x}$			
	(a)	Describe the transformation from g to h.	(2)	**L3**
	(b)	Determine the range of h.	(1)	**L1**
			[08]	

3.13

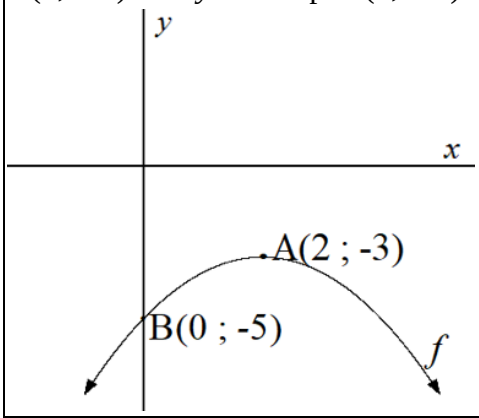
In the diagram below, *g* represents the function $g(x)=a^x, a>0$.
The graph of *h* is symmetrical to *g* about the *y*-axis. The point $B\left(2;\ 2\frac{1}{4}\right)$ lies on the curve of g and C is the *y*-intercept of both *g* and *h*.



3.13.1	Write down the range of *g*.	(1)	**L1**
3.13.2	Calculate the value of *a*.	(2)	**L2**
3.13.3	Write down the coordinates B', the image of B which lies on *h*.	(1)	**L1**
3.13.4	Determine the equation of *h* in the form $y=b^x$.	(2)	**L2**
3.13.5	Write down the coordinates of C.	(1)	**L1**
3.13.6	Hence, or otherwise, determine the value(s) of *x* for which $1<g(x)<2\frac{1}{4}$.	(2)	**L2**
3.13.7	B' is the reflection of B in the $y-$axis. Calculate the average gradient between B' and C.	(2)	**L2**
		[11]	

3.14

The diagram below shows the graph of a parabola *f*, with turning point A(2; −3) and *y*-intercept B(0; −5).



3.14.1	Show that the equation can be written as $y=-\frac{1}{2}x^2+2x-5$.	(5)	**L3**
3.14.2	Use the graph to determine the value(s) of *k* for which the equation $-\frac{1}{2}x^2+2x-5=k$ will have real and unequal roots.	(3)	**L3**
3.14.3	The parabola *f* is shifted vertically until the new *y*-intercept is at the origin. Determine the new turning point of the parabola.	(2)	**L2**
		[9]	

3.15 **DBE/FEB-MARCH 2016**

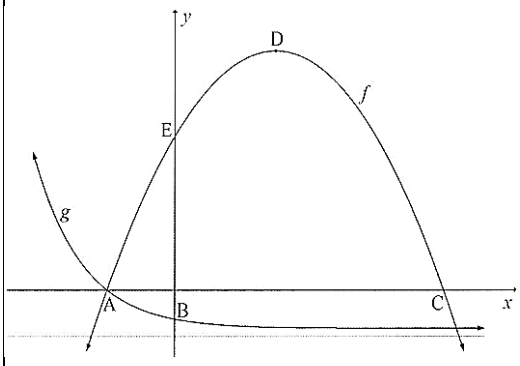
Given: $f(x)=2^x+1$

3.15.1	Determine the coordinates of the $y-$intercept of f.	(1)	**L1**
3.15.2	Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes.	(3)	**L2**
3.15.3	Calculate the average gradient of f between the points on the graph where $x=-2$ and $x=1$.	(3)	**L2**
3.15.4	If $h(x)=3f(x)$, write down an equation of the asymptote of h.	(1)	**L3**
		[08]	

3.16 **GRADE 11DBE NOV 2018**

The diagram below shows the graphs of $f(x) = -(x-3)^2 + 25$ and $g(x) = 2\left(\frac{1}{2}\right)^{x+1} - 4$.

Graph *f* cuts the *x*-axis at A and C, the *y*-axis at E and has a turning point at D. Graph *g* cuts the *x*-axis at A and the *y*-intercept at B.



3.16.1	Write down the equation of the asymptote of *g*.	(1)	**L1**
3.16.2	Write down the coordinates of D.	(2)	**L1**
3.16.3	Write down the range of *f*.	(1)	**L1**
3.16.4	Calculate the length of EB.	(4)	**L3**
3.16.5	Determine the values of *x* for which *f* is increasing.	(2)	**L2**
3.16.6	Calculate the average gradient between points A and B.	(4)	**L3**
3.16.7	Graph *t* is obtained by reflecting *g* about *x*-axis, Write down the range of *t*.	(2)	**L3**
3.16.8	If $p(x) = f(x) + 2$, Write down the the coordinates of the turning point of *p*.	(2)	**L2**
3.16.9	Determine the value of *k* for which the straight line $y = 2x+k$ will be a tangent to *f*.	(4)	**L4**
		[23]	

3.17 **HUDSON PARK HIGH SCHOOL JUNE 2018**

Given: $h(x) = 2.3^x - 6$

3.17.1	Sketch the graph of $h(x)$ clearly showing all asymptotes and intercepts.	(4)	**L2**
3.17.2	Is $h(x)$ an increasing or decreasing function?	(1)	**L1**
3.17.3	State the range of $h(x)$.	(1)	**L1**
3.17.4	If $h(x)$ is moved • 5 units vertically downwards; and • 4 units horizontally to the right to become $g(x)$. State the equation of $g(x)$ in *y*-form.	(2)	**L2**
		[8]	

3.18

$f(x) = \frac{4}{2-x} + 1$

3.18.1	Write down the equations of the asymptotes.	(2)	**L1**
3.18.2	Sketch the graph of $g(x)$ if $g(x) = -f(x)$	(4)	**L3**
		[06]	

3.19

rewrite the equation alongside in the form of: $f(x) = \frac{a}{x+b} + c$

3.19.1	$f(x) = \frac{x-7}{x+5}$	(3)	**L2**
3.19.2	$f(x) = \frac{2x-9}{2-x}$	(3)	**L2**
3.19.3	$f(x) = \frac{5-2x}{4-x}$	(3)	**L2**
		[09]	

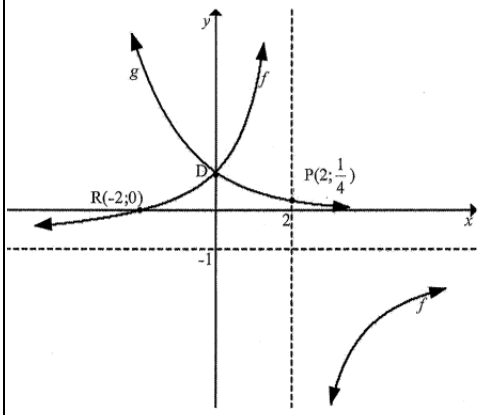
3.20 **KZN JUNE 2016**

The diagram below represents the graphs of $f(x)=\frac{a}{x+p}+q$ and $g(x)=t^x$.

f cuts the x-axis at R(–2 ; 0) and the y-axis at D.

$P\left(2;\frac{1}{4}\right)$ is a point on the graph of g.

f and g intersect at point D.

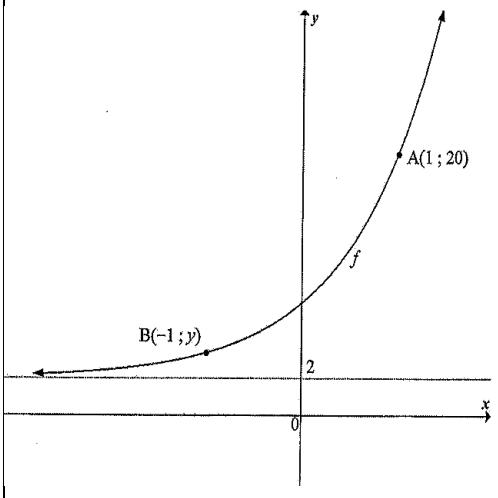


3.20.1	Write down the values of p and q.	(2)	**L2**
3.20.2	Determine the value of a.	(3)	**L2**
3.20.3	Determine the value of t.	(3)	**L2**
3.20.4	Calculate the average gradient of g between $x=-2$ and $x=2$.	(3)	**L3**
3.20.5	Write down the equation of the asymptote of g.	(1)	**L1**
3.20.6	Write down the coordinates of D.	(2)	**L1**
3.20.7	Determine the value(s) of x for which:		
	(a) $x.g(x)<0$.	(2)	**L2**
	(b) $g(x)\geq f(x)$.	(3)	**L3**
3.20.8	Determine the equation of the axis of symmetry of f that has a negative gradient.	(3)	**L2**
3.20.9	Determine the coordinate of R', The image of R about the line determined in QUESTION 3.15.8	(6)	**L4**
		[19]	

3.21 **DBE NOV 2016**

The sketch below is the graph of $f(x)=2.b^{x+1}+q$.

The graph of f passes through the points A(1; 20) and B(-1; y). The line $y = 2$ is an asymptote of f.



3.21.1	Show that the equation of f is $f(x)=2.(3)^{x+1}+2$.	(3)	**L3**
3.21.2	Calculate the y-coordinate of the point B.	(1)	**L2**
3.21.3	Determine the average gradient of the curve between points A and B.	(2)	**L2**
3.21.4	A new function h is obtained when f is reflected about its asymptote.	(2)	**L**
	(a) Determine the coordinates of A', the image of A which lies on h.	(2)	**L2**
	(b) Write down the range of h.	(1)	**L1**
	(c) Determine the equation of h.	(4)	**L4**
3.21.5	The graph of f has undergone a transformation to obtain $g(x)=2.(3)^x$.		
	(a) Describe the transformation that f has undergone in words.	(2)	**L2**
	(b) Hence, determine the range of g	(1)	**L1**
		[9]	

3.22 **KZN JUNE 2022**

Given: $h(x)=\left(\frac{1}{3}\right)^x+4$

3.22.1	Write down the equation of the asymptote of *h*.	(1)	**L1**
3.22.2	Draw a sketch of *h*, clearly indicating the asymptote and intercept with the axis.	(3)	**L2**

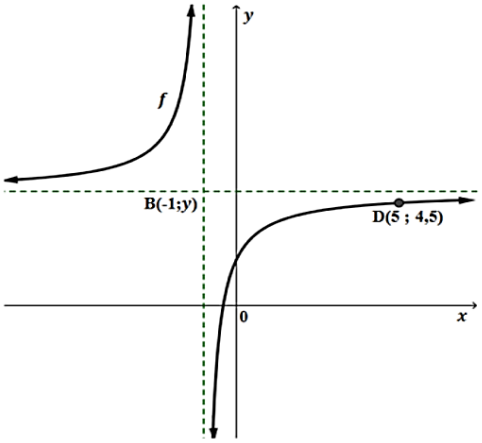
3.23 **DBE NOV 2017**

Given: $f(x)=\frac{-3}{x+2}+1$ and $g(x)=2^{-x}-4$.

3.23.1	Determine $f(-3)$.	(1)	**L1**
3.23.2	Determine x if $g(x)=4$.	(2)	**L2**
3.23.3	Write down the asymptotes of *f*.	(2)	**L1**
3.23.4	Write down the range of *g*.	(1)	**L1**
3.23.5	Determine the coordinates of the *x*- and *y*- intercepts of *f*.	(5)	**L3**
3.23.6	Determine the equation of the axis of symmetry of *f* which has a negative gradient. Leave your answer in the form $y=mx+c$.	(2)	**L2**
3.23.7	Sketch the graphs of *f* and *g* on the same system of axes. Clearly show ALL intercepts with the axes and asymptotes.	(6)	**L2**
3.23.8	If it is further given that $f(-1)=g(-1)$, Determine the values of *x* for which $g(x)\geq f(x)$.	(2)	**L4**

3.24

Sketched below is the graph of $f(x)=\frac{-3}{x+p}+q$. The asymptote of *f* intersect at $B(-1;y)$ and $B(5;4{,}5)$ is a point on *f*.



3.24.1	Write down the value of *p*.	(1)	**L1**
3.24.2	Prove that the value of $q=5$.	(2)	**L2**
3.24.3	Calculate the *x*-intercept of f. .	(2)	**L2**
3.24.4	Write down the equation of the vertical asymptote of *h*, if $h(x)=f(x+4)$.	(2)	**L2**
3.24.5	One of the axes of symmetry of f is an increasing function. Determine the equation of this axis of symmetry.	(3)	**L2**
3.24.6	Determine the values of *x* for which $f(x)>x+6$.	(2)	**L3**
		[12]	

3.25

Draw the graphs of the following functions, clearly indicating all intercepts (*x* and *y*) on the same system of axes

3.25.1	$f(x)=\frac{-3}{3-x}+1$ and $g(x)=-x^2+3x+4$	(4)	**L2**
3.25.2	$f(x)=2^x+1$ and $t(x)=-2x-5$	(4)	**L2**
3.25.2	$g(x)=4-x^2$ and $f(x)=-2x+2$	(4)	**L2**
3.25.3	$g(x)=\left(\frac{1}{5}\right)^x$ and $g(x)=-x^2-1$	(4)	**L2**
3.25.4	$j(x)=3^{-x}$ and $g(x)=2x^2-4x-6$	(4)	**L2**
		[16]	

TOPIC	4. FINANCE, GROWTH AND DECAY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

TERMINOLOGY:

- **Inflation** – refers to the rate at which prices of goods and services increase over time.
- **Population Growth** – refers to the increase in the number of individuals in population over time.
- **Reducing Balance** – is the method of depreciation that is calculated at a fixed percentage rate of the book value of the assets.
- **Simple growth-** refers to a type of growth that is calculated on the initial/original amount of investment or population.
- **Simple decay –** refers to whereby the value of an asset is reduced by a constant amount each year, which is calculated on the initial amount.
- **Compound growth-** refers to a type of a growth that is calculated on every year amount of investment or population based on recent accumulation.
- **Compound decay -** refers to a type of a growth that is calculated on every year amount of investment or population based on recent depreciations.
- **Nominal interest rate** is the quoted annual interest rate (this interest rate excludes the effect of inflation on price).
- **Effective interest rate** – refers to the actual rate of interest that is obtained.
- **Book value –** Refers to the value of an item after depreciation has taken place.
- **Scrap value –** Refers to the value of an item after its useful life.

SIMPLE GROWTH AND COMPOUND GROWTH $(A > P)$

$A \rightarrow$ Final amount. $P \rightarrow$ P amount. $i \rightarrow$ Interest rate. $n \rightarrow$ Period.

Simple growth	Compound growth
$A = P(1 + i \times n)$	$A = P(1 + i)^n$

SIMPLE AND COMPOND DECAY $(A < P)$

- **Simple decay** is associated with a decreasing STRAIGHT-LINE graph over years and is also called **linear depreciation method.**
- **Compound decay** is associated with a decreasing EXPONENTIAL graph and is also called **reducing balance method**.

$A \rightarrow$ Final amount $P \rightarrow$ Principal amount $i \rightarrow$ Interest rate $n \rightarrow$ Period

Linear depreciation method	Reducing balance method
$A = P(1 - i \times n)$	$A = P(1 - i)^n$

DIFFERENT COMPOUNDING PERIODS

In grade 11, it is possible to earn interest anytime so when interest is not earned once a year, in formula, we need to adjust ***i*** and ***n*** to match with given compounding periods. The table below shows different compounding periods.

Compounding periods	Annually (every year)	Half-yearly (every 6 months)	Quarterly (every 3 month)	Monthly (every 12 months)	Weekly (every 52 weeks)
Value to be used	$m=1$	$m=2$	$m=4$	$m=12$	$m=52$
Interest rate	i	$\frac{i}{2}$	$\frac{i}{4}$	$\frac{i}{12}$	$\frac{i}{52}$
Period (number of periods)	n	$2n$	$4n$	$12n$	$52n$

$A \rightarrow$ Final amount $P \rightarrow$ Principal amount $i \rightarrow$ Interest rate $n \rightarrow$ Period

Compound growth	**Compound decay**
$A = P\left(1+\frac{i}{m}\right)^{m\cdot n}$	$A = P\left(1-\frac{i}{m}\right)^{m\cdot n}$

These above formulas can also be used for sake of convenience to learners**.**

NOMINAL AND EFFECTIVE INTEREST

- **Effective interest rate** is noticed where compounding period is taken into consideration. The stated period and compounding period are the same.
- **Nominal interest rate** is an annual rate which financial institutions quote.

To determine the annual effective rate, we use the following formula:

$$1+i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^{m}$$

$i_{eff} =$ effective interest rate $m =$ compounding periods

$i_{nom} =$ Nominal interest rate

When working with different compounding periods use the formula:

$$\left(1+\frac{i_{new}}{n}\right)^{n} = \left(1+\frac{i_{new}}{m}\right)^{m}$$

TIMELINES

- Timelines are useful in financial Mathematics when there are several deposits and withdrawals, as well as possible changes in interest rates.
- Timelines provides a visual summary of all the information in an investment fund.

REVISION QUESTIONS

Simple growth and compound growth

4.1 Nomzamo wants to invest R1000 in a savings account. This type of an account will give 10% p.a. simple interest. How much money will Nomzamo have after 5 years? (2) **L1**

4.2 A large aviary keeps birds. There were 250 birds 3 years ago. Recently the birds are 316. What was the compounded population growth (in % form). (2) **L2**

4.3 Nhlathi invested R20 000 for 36 months at an interest rate of $x\%$ p.a. compounded annually. His final amount after 3 years was R26 620. Calculate the interest rate, to the **nearest whole number**. (2) **L2**

4.4 Yoli wishes to receive R120 000 in 3,25 years' time. How much money must she invests in an account that offers 8% p.a. compounded monthly. (3) **L3**

4.5 A school buys tablets at a total cost of R140 000. If the average rate of inflation is 6,1% per annum over the next 4 years, determine the cost of replacing these tablets in 4 years' time. (3) **L3**

Simple and compound decay

4.6 Mrs Smith bought a second hand car with R250 000 and the car depreciates at 13% p.a. compounded annually. Determine the scrap value of the car in 8 years' time. (2) **L1**

4.7 An amount of a construction vehicle worth of R150 000 after 6 years' time, depreciated at 12,5% p.a. on the straight-line method. Determine the cost price of vehicle. (3) **L2**

4.8 A business installs a server for R500 000. The value of the server depreciates at 20% per annum according to the diminishing-balance method. Calculate the scrap value of the server at the end of 6 years. (3) **L1**

4.9 A car, costing R198 000 has a book value of R102 755, 34 after 3 years. If the value of the car depreciates at $r\%$ p.a. on a reducing balance, calculate r. (4) **L2**

Different compounding periods

4.10 How much will R25 000 worth in 6 years' time in an account that offers 14% p.a. compounded monthly? (3) **L2**

4.11 How much will Qiniso receive in 3 years if he:…

(a) invests R2000 in an account that offers 12% p.a. compounded monthly? (2) **L2**

(b) invests R8 050 in an account that offers 14,25% p.a. compounded quarterly? (3) **L3**

(c) invests R452 000 in an account that offers 9,5% p.a. compounded half-yearly? (3) **L3**

Nominal and effective interests

4.12 Convert 18% p.a compounded half-yearly to an effective interest rate. (2) **L2**

4.13 Convert the following effective interest rates to nominal interest rates.

(a) 8,5% p.a. to be compounded monthly. (2) **L2**

(b) 33% p.a. to be compounded bi-annually. (2) **L3**

4.14 Calculate the effective interest rate if the interest rate is 9,8%, compounded monthly. (2) **L2**

4.15 Determine the effective interest rate if an investment earns interest at a nominal interest rate of 11,5% per annum, compounded quarterly. (3) **L2**

4.16 An investment earns interest at a rate of 7% per annum, compounded semi-annually. Calculate the effective annual interest rate on this investment. (3) **L3**

Timelines

4.17 A savings account is opened with a deposit of R9 400 and two years later a further R5 800 is added to the savings account. Five years after the saving account was opened, another R12 600 is deposited into the account. One year after this, R1 500 is withdrawn. The interest paid on the savings is 9% p.a. compounded monthly. Calculate the total amount saved at the end of the nine years. (5) **L4**

4.18 A savings account was opened with an initial deposit of R24 000. 18 months later R7 000 was withdrawn from the account. Calculate how much money will be in the savings account at the end of 4 years if the interest rate was 10,5% p.a. compounded monthly. (4) **L3**

4.19 Siyabonga deposited R25 000 into a savings account with an interest rate of 18% p.a, compounded monthly. Siyabonga withdrew R8 000 from the account 2 years after depositing the initial amount. He deposited another R4 000 into this account $3\frac{1}{2}$ years after the initial deposit. What amount will Siyabonga have 5 years after making the initial deposit in this account? (6) **L4**

MIXED QUESTIONS

4.20 A machine costs R25 000 in 2016. Calculate the book value of the machine after 4 years if it depreciates at 9% p.a according to the reducing balance method. (3) **L2**

4.21 Exactly 8 years ago, Tashil invested R30 000 in an account earning 6,5% per annum, compounded monthly.

(a) How much will he receive if he withdraws his money today? (3) **L2**

(b) Tashil withdrew R10 000 three years after making the initial deposit and re-invested R10 000 five years after making the initial deposit. (5) **L4**
Calculate the difference between the final amount Tashil will now receive after eight years and the amount he would have received had there not been any transactions on the account after the initial deposit.

4.22 A company buys a front-end loader at a cost of R1 800 000. The book value of the loader at the end of 4 years is R720 000. Calculate the annual rate of depreciation on the reducing balance method. (5) **L2**

4.23 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5) **L3**

4.24 Nhlanhla made an initial deposit of R20 000 into an investment account that paid interest at the rate of 7,2% p.a., compounded quarterly. After two years, the interest changed to 7,8% p.a compounded monthly. Four years after his initial deposit, Nhlanhla withdrew R2 500 from his investment.

(a) Calculate how much Nhlanhla had in this investment account two years after the initial deposit was made? (3) **L3**

(b) How much will the investment be worth 7 years after the initial deposit was made? (4) **L3**

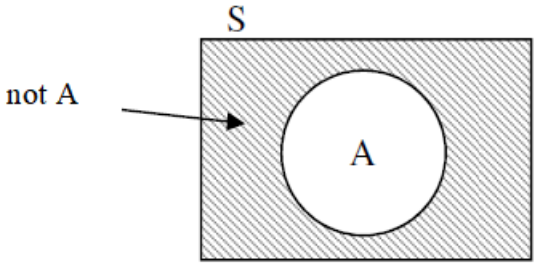
TOPIC	5. PROBABILITY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

- Mark Weighting : Term 3 common test, $\frac{20 \pm 3}{75}$

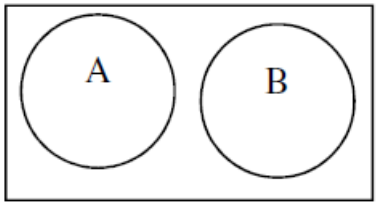
  : November final paper, $\frac{25 \pm 3}{150}$

- $P(A) = \frac{n(E)}{n(S)}$, where $0 \le P \le 1$. This simple means that probability **canno**t be negative or above 1.
- General formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where 'A and B' are the events that happen at the same time.
- Complimentary events: $P(A) + P(\text{not } A) = 1$. It simple means that event A and not A cannot happen at the same time.

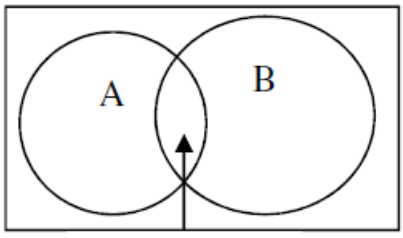


- Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$. Event A and B cannot happen at the same time.

  Events A and B are said to be disjoint.



- Not Mutually Exclusive Events: $P(A \text{ and } B) \neq 0$



A and B

- Independent events: The outcome of the second event is not affected by the outcomes of the first event. For independent event:

  $P(A \text{ and } B) = P(A).P(B)$

  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

  $P(A \text{ or } B) = P(A) + P(B) - P(A).P(B)$

Mutually Exclusive And Independent Events

DBE Nov 2016 (Gr 11, Q 9.1)

5.1 Given: $P(A) = 0,2$

$P(B) = 0,5$

$P(A \text{ or } B) = 0,6$ where A and B are two different events.

5.1.1 Calculate $P(A \text{ and } B)$. (2) **L1**

5.1.2 Are events A and B independent? Show your calculations. (3) **L2**

DBE Nov 2015 (Gr 11, Q 9.1)

5.2 Given: $P(A) = 0,6$

$P(B) = 0,3$

$P(A \text{ or } B) = 0,8$ where A and B are mutually exclusive events.

Are events A and B mutually exclusive? Justify your answer with appropriate calculations and/or a diagram. (4) **L2**

DBE Nov 2018 (Gr 11, Q 9.2)

5.3 For any two events, A and B, it is given that $P(A \text{ and } B) = 0,12$; $P(A \text{ or } B) = 0,83$ and $P(B) = 4P(A)$.

5.3.1 Are events A and B mutually exclusive? Justify your answer. (1) **L1**

5.3.2 Calculate $P(B)$. (4) **L2**

5.3.3 Calculate $P(\text{not } A)$ (2) **L2**

DBE Nov 2014 (Gr 11, Q 11)

5.4. Given: $P(A) = 0,12$

$P(B) = 0,35$

$P(A \text{ or } B) = 0,428$

Determine whether events A and B are independent or not. Show ALL relevant calculations used in determining the answer. (4) **L2**

DBE Nov 2018 (Gr 12, Q 12.1)

5.5 Given: $P(A) = 0,45$; $P(B) = y$ and $P(A \text{ or } B) = 0,74$

Determine the value(s) of y if A and B are mutually exclusive. (3) **L3**

DBE Nov 2015 (Gr 12, Q 11.1)

5.6 For two events, A and B, it is given that:

$P(A) = 0,2$

$P(B) = 0,63$

$P(A \text{ and } B) = 0,126$

Are events, A and B, independent? Justify your answer with appropriate calculations. (3) **L2**

Contingency Tables

KZN Sep 2016 (Gr 11, Q 1)

5.7 A travel agent did a survey amongst his clients as to which type of holiday they prefer. The table below shows the results of this survey.

	GAME RESERVE	SEA	DRAKENSBERG	TOTAL
MALE	250	70	a	500
FEMALE	150	b	160	c
TOTAL	d	170	e	910

5.7.1 Write down the values of a, b, c, d and e. (2) **L1**

5.7.2 If a client is selected at random, what is the probability that this client will:

(a) prefer visiting a game reserve? (1) **L1**

(b) be a female? (1) **L1**

(c) be a female and prefer visiting a game reserve? (1) **L1**

(d) be a female or prefer visiting a game reserve? (2) **L2**

5.7.3 Consider the following two events:

Event A: a client is a female

Event B: a client prefers visiting a game reserve when going on holiday.

Are events A and B independent? Motivate your answer with the necessary calculations. (3) **L3**

DBE Nov 2018 (Gr 11, Q 9.1)

5.8 On a flight, passengers could choose between a vegetarian snack and chicken snack. The snacks selected by the passengers were recorded.

SNACK	MALE	FEMALE	TOTAL
VEGETARIAN	12	20	32
CHICKEN	55	63	118
TOTAL	67	83	150

Was the choice on snacks of this flight independent of gender? Motivate your answer with the necessary calculations. (5) **L3**

DBE Feb/ March 2016 (Gr 12, Q10.1)

5.9 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	b	c	80
Fruit juice	d	e	20
TOTAL	60	100	a

5.9.1 Write down the value of a. (1) **L1**

5.9.2 What is the probability that a randomly selected passenger is a male? (2) **L1**

5.9.3 The events of a passenger choosing coffee and being a male are independent of being a male, calculate the value of b. (4) **L3**

Adapted From June 2018 (Gr 12, Q 11)

5.10 A survey on the preference of exercise was conducted among 140 people in two age groups. The information is summarized below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
older than 35 years	b	21	d	60
TOTAL	21	49	70	140

5.10.1 Calculate the probability that an older than 35 years person prefers running. (2) **L1**

5.10.2 The event of preferring to play tennis and age are independent of each other, calculate the value of a. (4) **L3**

Venn Diagrams

DBE Nov 2017 (Gr 12, Q 10)

5.11 A survey was conducted among 100 grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cellphones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

5.11.1 Draw a Venn diagram to illustrate the information. (4) **L2**

5.11.2 Calculate the value of x. (2) **L2**

5.11.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2) **L2**

DBE Nov 2011 (Gr 12, Q 6)

5.12 Complaints about a restaurant fell into three main categories: the menu (M), the food (F) and the service (S). In total 173 complaints were received in a month. The complaints were as follows:

- 110 complained about the menu.
- 55 complained about the food.
- 67 complained about the service.
- 20 complained about the menu and the food, but not the service.
- 11 complained about the menu and the service, but not the food.
- 16 complained about the food and the service, but not the menu.
- The number who complained about all three is unknown.

5.12.1 Draw a Venn diagram to illustrate the above information. (6) **L2**

5.12.2 Determine the number of people who complained about ALL THREE categories. (3) **L2**

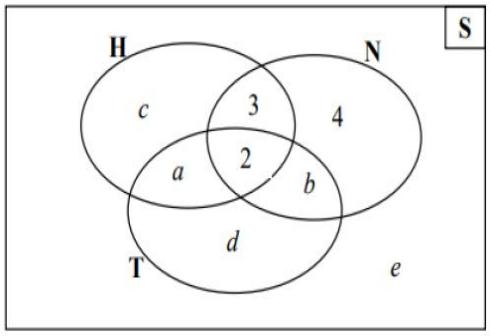
5.12.3 Determine the probability that a complaint selected at random from those received, complained about AT LEAST TWO of the categories (that is menu, food and service). (3) **L3**

DBE Exemplar 2013 (Gr 11, Q 12.1)

5.13 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 leaners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

5.13.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e.



(5) **L2**

5.13.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1) **L1**

5.13.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1) **L1**

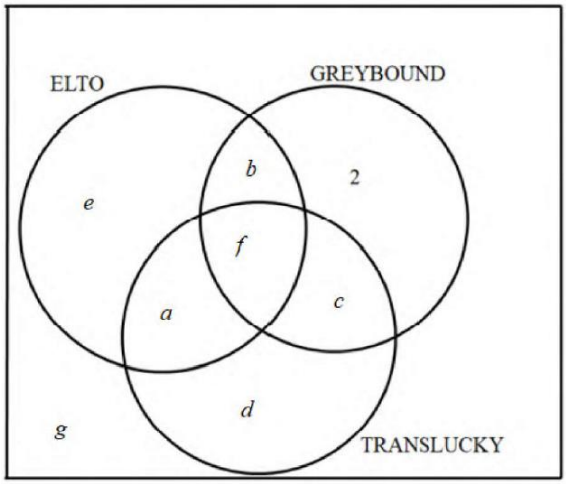
5.13.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1) **L2**

KZN Sep 2023 (Gr 12, Q 12)

5.14 A group of 40 people was asked which bus company they liked to travel by: Elto, Greybound or Translucky bus company.

- 3 people liked travelling by all 3 bus companies.
- 7 people liked to travel by Trunslucky and Elto.
- 3 people did not like using any of the bus companies.
- The probability of a randomly selected person liking to travel by Greybound and Elto is $\frac{2}{5}$.
- The probability of a randomly selected person liking to travel by Greybound and Translucky is $\frac{1}{5}$.
- The probability of a randomly selected person liking to travel by Translucky only is $\frac{1}{10}$.
- The probability of a randomly selected person liking to travel by Elto is $\frac{13}{20}$.

The partially completed Venn diagram drawn below represents the given information.



5.14.1 Use the given information to determine the values of a, b, c, d and e. (5) **L3**

5.14.2 How many people like to travel by Greybound bus company? (1) **L2**

5.14.3 Calculate the probability of randomly selected person liking to travel by only one bus company. (2) **L2**

Tree Diagrams

DBE Exemplar 2014 (Gr 12, Q 11.2)

5.15 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

5.15.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4) **L2**

5.15.2 What is the probability that a yellow ball will be chosen from Bag A? (1) **L1**

5.15.3 What is the probability that a pink ball will be chosen? (3) **L3**

DBE Feb/ March 2017 (Gr 12, Q 11)

5.16 The success rate of the Fana soccer team depends on a number of factors. The fitness of the player is one of the factors that influence the outcome of the match.

- The probability that all the players are fit for the next match is 70%.
- If all the players are fit to play the next match, the probability of winning the next match is 85%.

If there are players that are not fit to play the next match, the probability of winning the match is 55%.

Based on fitness alone, calculate the probability that the Fana soccer team will win the next match. (5) **L3**

FS Sep 2018 (Gr 12, Q 11.2)

5.17 The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny, the probability that Pitso plays soccer is $\frac{4}{5}$. If it is not sunny, the probability that Pitso plays soccer is $\frac{2}{5}$. Determine the probability that Pitso does not play soccer. (4) **L3**

DBE Nov 2015 (Gr 12, Q 11,3)

5.18 There are t orange balls and two yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to the bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6) **L4**

KZN Nov 2023 (Gr 11, Q 9.2)

5.19 A picnic basket contains x apples and 8 bananas. A fruit is selected at random and eaten. Thereafter, another fruit is selected at random and eaten. The probability of selecting two different fruits, in any order, in the first two selections is $\frac{48}{95}$.

Calculate the total number of apples in the basket. (5) **L4**

DBE Exemplar 2013 (Gr 11, Q 12.2)

5.20 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy. At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at school does Mathematical Literacy is 65%.
Determine the probability that a learner selected at random from this school does Mathematics. (6) **L3**

EC Sep 2022 (Gr 12, Q 11)

5.21 A bag contains only two colours of tennis balls, red and green, in the ratio 1:3. Two balls are picked at random, one after the other, without replacement. The probability of picking first red a red ball and second green ball is $\frac{1}{5}$. Calculate the number of balls in the bag. (5) **L4**

TOPIC:	6. STATISTICS

GUIDELINES, SUMMARY NOTES, & STRATEGIES

- **Histograms**
- **Frequency polygons**
- **Ogives (cumulative frequency curves)**
- **Variance and standard deviation of ungrouped data**
- **Symmetric and skewed data**
- **Identification of outlier**

Terminology:

- **Discrete data** – Data that can have definite values and usually collected by counting objects (for example, number of cars, value on a dice)
- **Continuous data** – Data that can have any value and collected by measurement given to a suitable degree accuracy (weight, height, time).
- **Sample** – data is collected on a specified set from a large group of elements.
- **Ungrouped data** – A set of random data elements gathered for analysis.
- **Grouped data** – Data elements aggregated into different classes, groups, or intervals.

Measures of central tendency – **Single** numbers around which all data items seem to be spread.

- The **Mean** (**Average**) – The sum of all the data values divided by the number of data values.
  $\bar{x} = \frac{\sum x}{n}$ or $\bar{x}_{est} = \frac{\sum f.x}{\sum f}$, *where f is the frequency and x is the midpoint of the class interval*
- The **Median** – the **middle value** after the data set has been arranged in ascending order.
  position of the median $Q_2 = \frac{1}{2}(n+1)$
- The **Mode –** The most common number or value that occurs the most often. The mode is the vale with the highest frequency.

Measures of dispersion – numbers that describe the spread of the data

- **The Range** – The difference between the maximum and the minimum data values in a given data set
- **The Inter-Quartile-Range (IQR)** – is the difference between the third and first quartiles i.e.
  $IQR = Q_2 - Q_1$
- **Standard Deviation (σ)** – is a measure of how dispersed data is around the mean. The square of the standard deviation is the variance (σ^2)

Quartiles – numbers that **divide** the data into **quarters** in an **ordered data set.**

- Lower quartile, (Q_1), is the data item below which a quarter of the data lies in an ordered data set.

  Position of the lower quartile $Q_1 = \frac{1}{4}(n+1)$

- Upper Quartile (Q_3) – is the data item above which a quarter of the data lies in an ordered data set

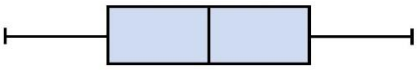
  position of the upper quartile $Q_3 = \frac{3}{4}(n+1)$

FIVE NUMBER SUMMARY – Five numbers that separate a data set into quarters

1. Minimum value
2. Lower Quartile (Q_1)
3. Median (Q_2)
4. Upper quartile (Q_3)
5. Maximum value.

Box-and-whisker Diagram/Plot (drawn using the five number summary)

- When analysing the distribution of data in a given set:
  - If mean = median, the distribution is symmetric
  - If mean > median, then the distribution is positively skewed.
  - If mean < median, then the distribution is negatively skewed.

Negatively skewed (Skewed left)	Symmetric	Positively Skewed (Skewed right)
		
mean < median	mean = median	mean > median

- In a symmetrical data set approximately 68% of the data will fall within one standard deviation of the mean
- Outliers – data items that are a lot bigger or smaller than the rest of the elements in the data set.

They are determined as follows:

- Lower outliers are numbers $< Q_1 - 1.5 \times IQR$
- Upper outliers are numbers $> Q_1 + 1.5 \times IQR$

Graphical representations:

- Histogram – represents grouped as condensed bars whose widths and length represents class intervals and frequency respectively.
- Ogive (Cumulative Frequency Curve) – an *S*-shaped smooth curve drawn by drawn by plotting upper limits of class intervals of a grouped data against cumulative frequency of the set.

ACTIVITIES

Grade 11 SEPT 2022 (KZN)

6.1 The data below shows the distance (in kilometres) travelled by cyclists on each day of a twelve-day cycle:

82	89	94	100	114	121
123	128	128	132	135	140

Determine:

6.1.1 The mean distance travelled per day (2) **L1**

6.1.2 The standard deviation of the data (1) **L1**

6.2 On how many days do the cyclists travel a distance that is outside of one standard deviation from the mean? Show all your calculations. (3) **L2**

The 5-number summary for the data is as follows:

(82 ; 97 ; 122 ; 130 ; 140).

6.2.1 Draw a box-and-whisker diagram for this set of data. (3) **L1**

6.2.2 Comment on the skewness of the data. (1) **L1**

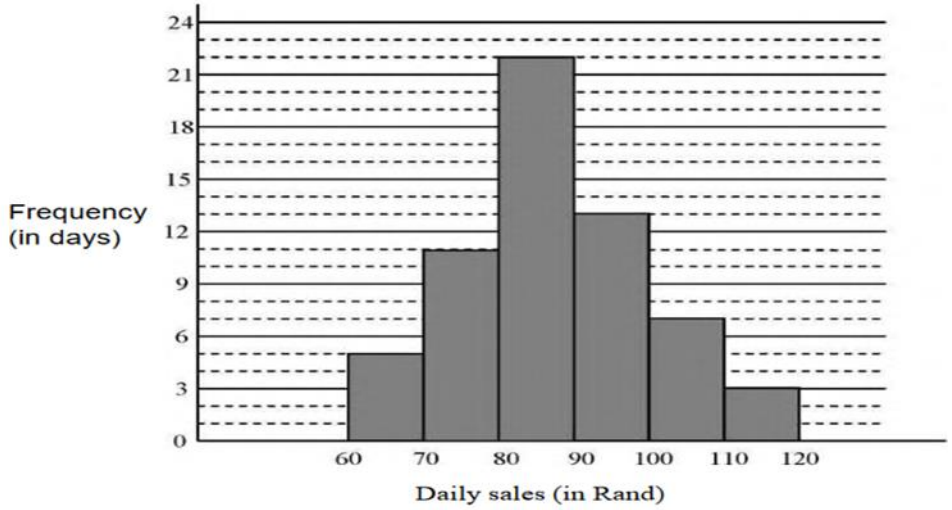
6.3 Before the start of each day's section of the tour, the cyclists had an additional warmup ride of $x$km. If this is also taken into account, write down, in terms of x (where applicable)

6.3.1 the mean distance travelled per day (2) **L3**

6.3.2 the standard deviation (1) **L2**

[13]

6.4 A street vendor has kept a record of sales for November and December 2007. The daily sales in Rand is shown in the histogram below.



6.4.1 Complete the cumulative frequency table for the sales over November and December. (3) **L2**

6.4.2 Draw an ogive for the sales over November and December. (3) **L2**

6.4.3 Use your ogive to determine the median value for the daily sales. (1) **L1**

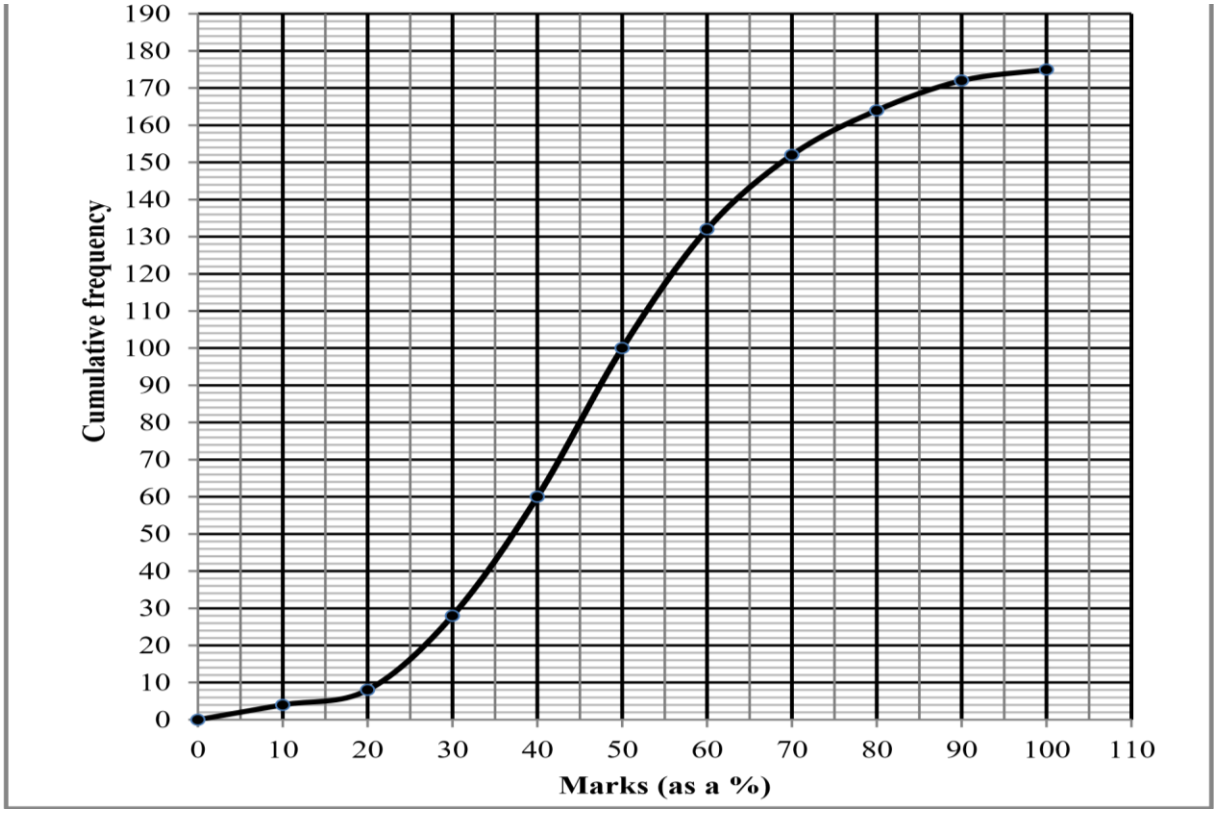
6.4.4 Estimate the interval of the upper 25% of the daily sales.. (2) **L2**

[09]

BDE NOVEMBER 2020(2) (GRADE 12)

6.5 Learners from various schools wrote an aptitude test in order to qualify for a bursary. Their marks (as a percentage) are represented in the ogive (cumulative frequency graph) below:

OGIVE



6.5.1 How many learners wrote the test? (1) **L1**

6.5.2 Write down the modal class of the data. (1) **L2**

6.5.3 The minimum mark to qualify for a bursary is 75%. How many learners qualified for a bursary? (2) **L2**

The table below shows the marks that 15 learners from one particular school obtained in the aptitude test.

Marks (as a %)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92

Calculate the:

6.5.4 mean mark obtained by these learners. (2) **L1**

6.5.5 standard deviation of these learners' marks. (1) **L1**

6.5.6 number of these learners whose marks lie more than one standard deviation above the mean. (2) **L2**

6.5.7 The final Grade 11 marks (as a percentage) obtained by a group of learners was analysed. The one standard deviation interval about the mean was calculated as (82,7 ; 94,1). Calculate the standard deviation for the final Grade 11 marks. (3) **L3**

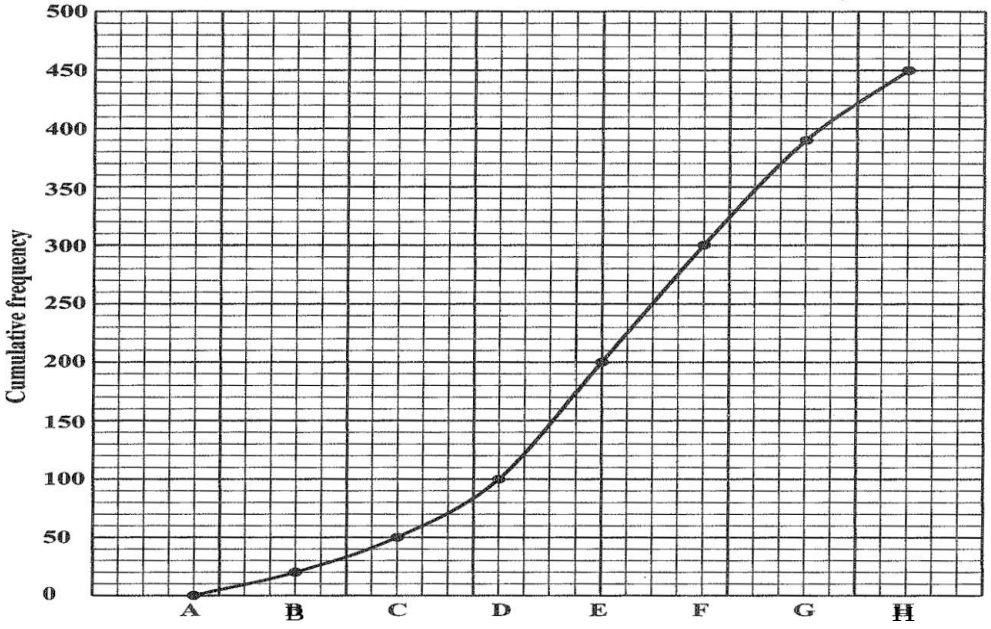
[12]

NW MATHS GRADE 12 SEPT 2022

6.6 A survey was conducted among a number of learners who were using the same cellphone company to establish the number of WhatsApp messages that they sent during a day. The results were summarised in the partially completed cumulative frequency graph (ogive) below.



Cumulative Frequency Graph (Ogive)



Number of WhatsApp messages sent during a day

The data shown in the cumulative frequency graph is also summarised in the partially completed frequency table below:

WhatsApp messages sent	**Frequency**
$40 < x \leq 80$	20
$80 < x \leq 120$	30
$120 < x \leq 160$	**Q**
$160 < x \leq 200$	**R**
$200 < x \leq 240$	100
$240 < x \leq 280$	90
$280 < x \leq 320$	60

6.6.1 Write down the value of A, a marked point on the horizontal axis of the cumulative frequency curve. (1) **L1**

6.6.2 How many learners participated in this survey? (1) **L1**

6.6.3 Calculate the values of Q and R, the frequencies in the frequency table above. (3) **L2**

6.6.4 Estimate the mean number of WhatsApp messages sent per day. (2) **L3**

6.6.5 Calculate the interquartile range. (3) **L3**

6.6.6 The cellphone company wants to introduce a new rule where you are not allowed to send more than 200 WhatsApp messages per day. If this rule is applied to this group of learners on the existing data, how would the standard deviation for the data be affected? Motivate your answer. (2) **L4**

[10]

MP GRADE 12 TRIAL EXAM 2023 P2

6.7 The table below shows the masses (in kg) of 15 randomly chosen weight lifters of a certain gymnasium.

71	83	88	91	92	92	95	97	104	108
109	110	111	115	129					

6.7.1 Calculate the mean mass of the weight lifters. (2) **L1**

6.7.2 Calculate the standard deviation of the masses of the weight lifters. (1) **L1**

6.7.3 What percentage of weight lifters fall in the feather weight division, if the criteria is that your mass must be below one standard deviation below the mean? (3) **L2**

6.7.4 If all the weight lifters lose 3kg, what will be the new:

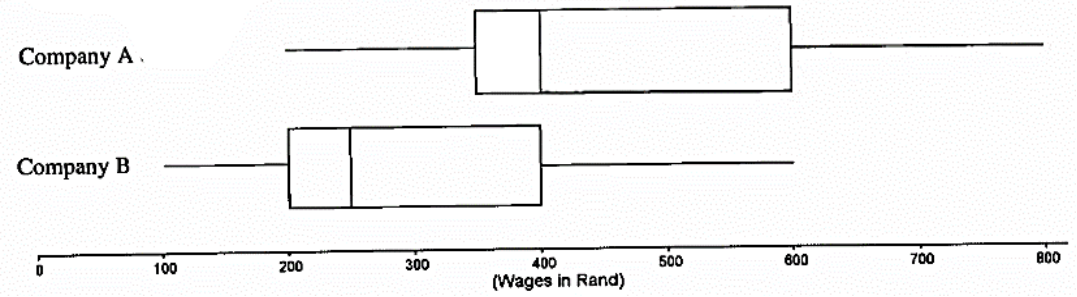
(a) mean mass of the weight lifters (1) **L2**

(b) standard deviation of the masses (1) **L3**

[08]

MP GRADE 12 TRIAL EXAM 2023 P2

6.8 The box-and-whisker plots show the wages of workers (in Rand) at two companies for the same work. Both companies have 20 workers.



6.8.1 State whether the following statement is TRUE or FALSE:
All the workers at Company A earn more than 25% of the workers at Company B. (1) **L4**

6.8.2 Comment on the skewness of the data for company B (1) **L1**

6.8.3 Which company has the biggest range? Motivate your with the necessary calculations (2) **L3**

6.8.4 How many workers at company B earn more than R200? (2) **L3**

[14]

6.9 The following table represents the heights, in centimetres, of 120 boys in a school.

HEIGHT (cm)	FREQUENCY	CUMULATIVE FREQUENCY
$150 < x \leq 155$	4	4
$155 < x \leq 160$	22	**A**
$160 < x \leq 165$	56	82
$165 < x \leq 170$	32	114
$170 < x \leq 175$	6	**B**

6.9.1 Complete the cumulative frequency table ABOVE by writing down the value of A and B. (2) **L1**

6.9.2 Draw an ogive, to represent the information in the table. (4) **L2**

6.9.3 Determine, using the ogive, the five number summary. (5) **L2**

6.9.4 If the distribution of the data is represented by means of a box and whisker diagram, comment on the spread of the data. (1) **L1**

[12]

6.10 The following is a sample of weekly wages earned by ten people working for a small printing and design company

R2 250 R2 250 R3 000 R3 300 R3 300
R3 600 R3 900 R4 350 R4 350 R5 250

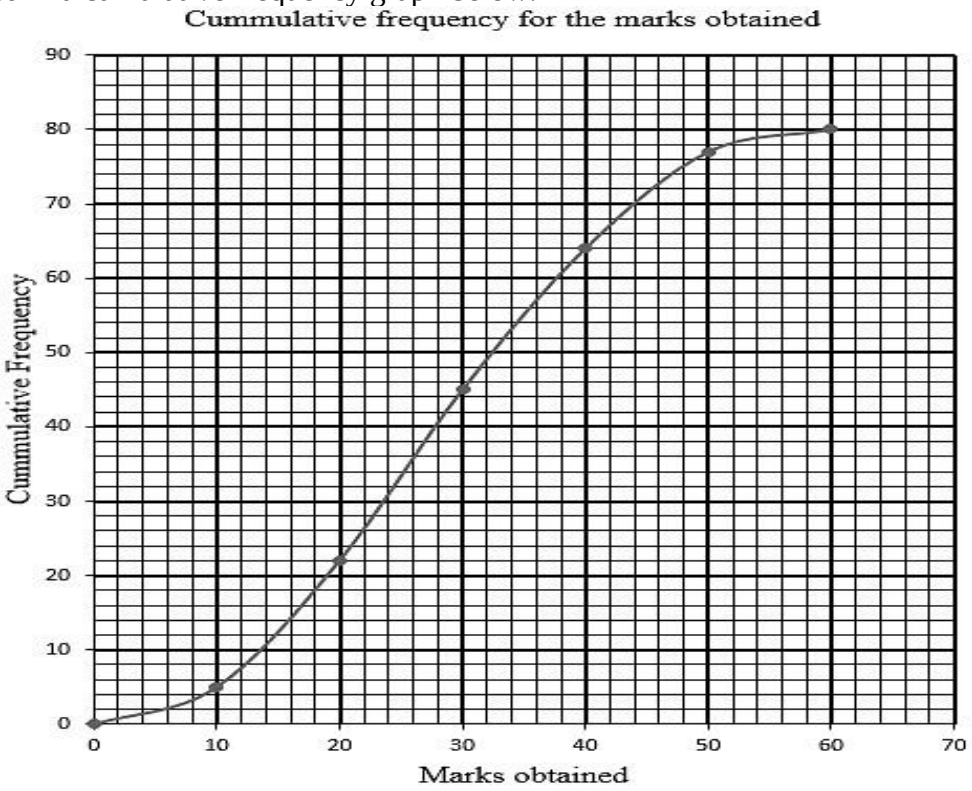
6.10.1 Calculate the mean weekly wage. (2) **L2**

6.10.2 Calculate the standard deviation of the weekly wage. (1) **L2**

6.10.3 Determine the percentage of workers which lie within ONE standard deviation of the mean. (4) **L3**

[07]

6.11 A group of learners wrote a standardised English test that was scored out of 60. The results were represented in a cumulative frequency graph below.



6.11.1 How many learners wrote the test? (1) **L1**

6.11.2 How many learners scored at least 20 out of 60? (2) **L2**

6.11.3 Using the graph, estimate the median test score. (2) **L2**

6.11.4 Complete the frequency table below.

Marks obtained	Frequency
$0 < x \leq 10$	
$10 < x \leq 20$	
$20 < x \leq 30$	
$30 < x \leq 40$	
$40 < x \leq 50$	
$50 < x \leq 60$	

(5) **L2**

6.11.5 Write down the modal group. (1) **L1**

[11]

6.12 Five data values are represented as follows: $2k$; $k + 1$; $k + 2$; $k - 3$; $2k - 2$

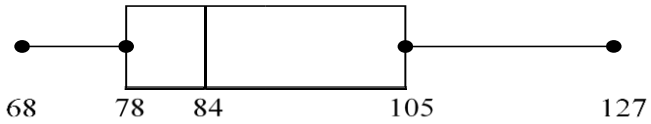
6.12.1 If the mean of the data set is 15, show that $k = 11$. (3) **L2**

6.12.2 Calculate the standard deviation for this data, rounded off to one decimal place (2) **L**

6.12.3 If t units is subtracted from each data value in the set, without further calculation, explain how the mean and standard deviation would be affected in terms of t. (2) **L3**

[07]

6.13 The box-and-whisker diagram below shows the distribution of the September examination marks of a group of 24 boys in Grade 11. The median is 84 and the mean is 87.



6.13.1 How many boys scored more than 105? (2) **L2**

6.13.2 In which direction are the marks skewed? (1) **L1**

6.13.3 Calculate the range of the data. (2) **L1**

6.13.4 On checking the answer book of the candidate who scored 127, an adding error is discovered and his mark is changed to 147. Determine the resulting value of the following measures:

(a) The median (1) **L2**

(b) The mean (2) **L3**

[07]

KZN JUNE P2 (2023)

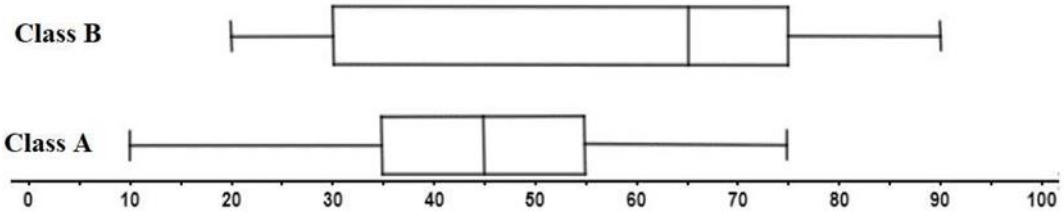
6.14 Seven scores for a test out of 50 marks were recorded. However, when recording the data, the statistician had some difficulty with his computer. He listed the following conditions that he had remembered.

Scores	a	b	c	d	e	f	g

- Maximum Value is 42
- Range is 35
- Median is 23
- The difference between the median and the upper quartile is 14
- The Inter-Quartile Range is 22
- $e = 2c$
- The mean is 25.

Determine the values of the scores. (Show all workings) (10) **L3**

6.15 The box and whisker diagrams below show the Mathematics results of class A and class B in the June Examination. It is also given that class B has a Median of 65%.



6.15.1 Which class had the top learners? (1) **L1**

6.15.2 Determine which class had the greatest Inter Quartile Range (IQR). (1) **L1**

6.15.3 What percentage of class A scored lower than 55%? (1) **L2**

6.15.4 If all the learners in class A were given an extra 5%, what would happen to the standard deviation of the marks in class A? (1) **L3**

6.15.5 Determine the semi-interquartile range of class B. (1) **L2**

[05]

6.16 Learners at Phambili High School travel from three different neighbourhoods, Neighbourhood A, B and C. The table below shows the number of learners from each neighbourhood, and their mean travelling times from home to school.

Neighbourhood	A	B	C
Number of learners	135	225	200
Mean travelling time (in min)	24	32	x

The mean travelling time for learners in Neighbourhood C is the **same** as the mean travelling time for all 560 learners
Calculate the mean travelling time for Neighbourhood C (4) **L4**

[04]

TOPIC	7. ANALYTICAL GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

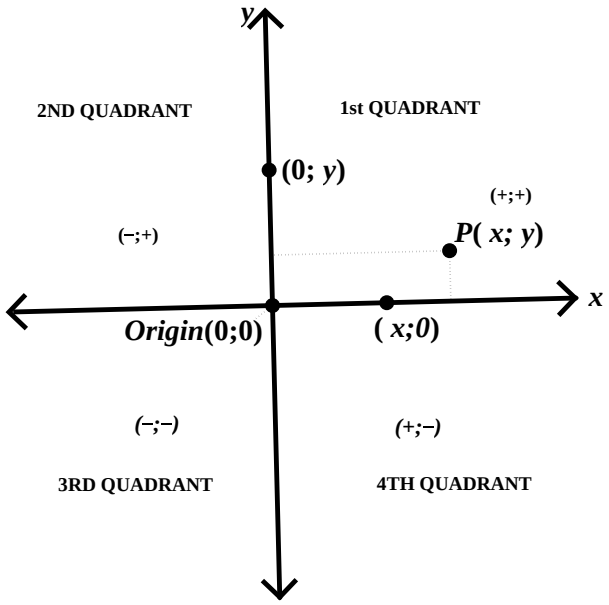
Analytical Geometry is the Geometry on the Cartesian plane,

- It is an algebraic approach to the study of Geometry "using coordinates".

In this topic, we will address the following concepts:

- The **distance** between two points (length of a line segment).
- The **midpoint** of a line segment.
- The **gradient** of a line.

Consider the Cartesian Plane:

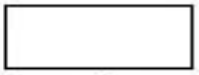
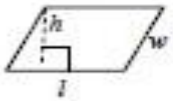
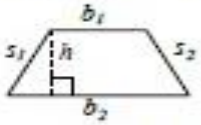
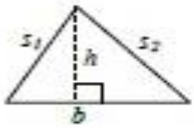
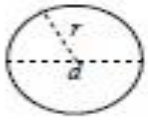




- Every point on the y – axis has $x = 0$
- Every point on the x - axis has $y = 0$
- In analytical geometry we represent geometric figures on the Cartesian plane and use coordinates of points to derive and prove important facts about these figures.



- Properties of triangles
- Pythagoras theorem
- Properties of Quadrilaterals
- Areas and perimeters of basic polygons

Square  $A = s^2$ $P = 4s$	**Rectangle** $A = lw$  $P = 2l + 2w$	**Parallelogram**  $A = lh$ $P = 2l + 2w$
Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$  $P = s_1 + s_2 + b_1 + b_2$	**Triangle**  $A = \frac{1}{2}bh$ $P = s_1 + s_2 + b$	**Circle**  $A = \pi * r^2$ $C = 2\pi * r$ or $C = \pi * d$

Formulae:

1. **The distance between two points**:

- The formula to calculate the distance /length of a line segment between points A and B, is given by:

$$\text{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

2. **The midpoint of a line segment**

- The midpoint of a line segment when given the coordinates of two points is given by:

$$m\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

- When given the midpoint and required to calculate the coordinate the formula is given by:

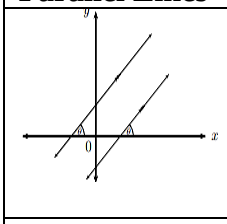
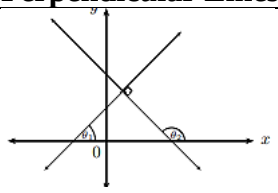
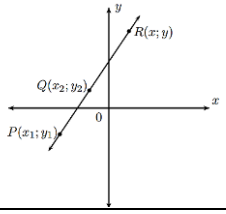
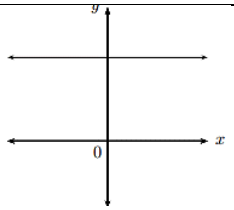
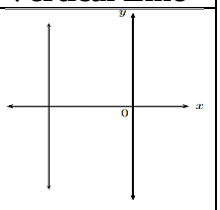
$$x = \frac{x_A + y_B}{2} \; ; \; y = \frac{y_A + y_B}{2}$$

3. **The gradient of a line**:

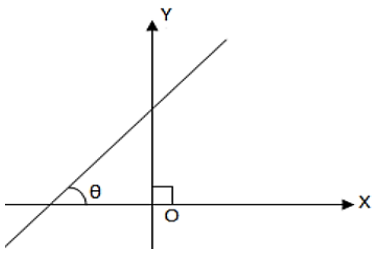
- The gradient of a line is given by:

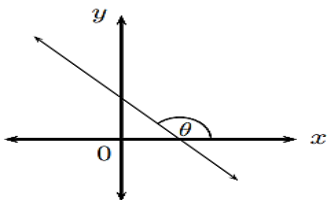
$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

More about gradient:

Parallel Lines	Perpendicular Lines	Collinear Points	Horizontal Line	Vertical Line
				
$m_1 = m_2$	$m_1 \times m_2 = -1$	$m_{PQ} = m_{QR} = m_{PR}$	$m = 0$	m is undefined

3. **THE INCLINATION OF A LINE**: $m = \tan\theta$





- The angle of inclination is $0° \le \theta \le 180°$
- Positive gradients (m) have an angle of inclination $0° < \theta < 90°$ (Acute angle)
- Negative gradients have an angle of inclination $90° < \theta < 180°$ (Obtuse angle)

2. **EQUATION OF A LINE:** $y-y_1=m(x-x_1)$ OR $y=mx+c$ / $y=ax+q$

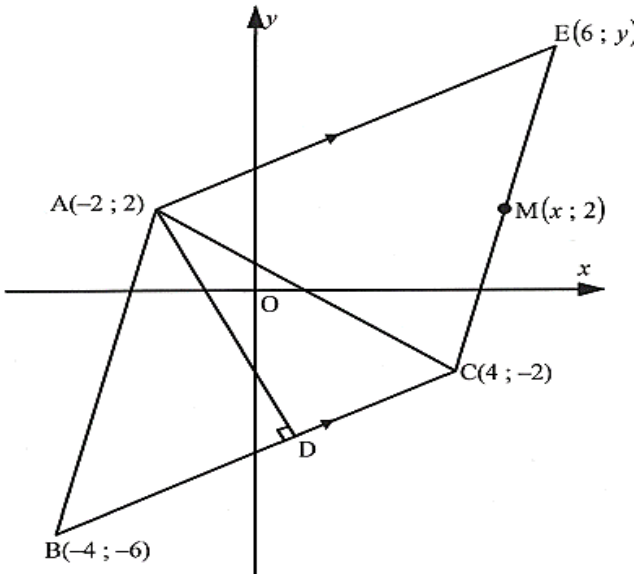
Determine equation of a line when given:

- The gradient and the y-intercept
- The gradient and one point on the line
- Line passing through two points
- Line through one point and parallel or perpendicular to a given line

ACTIVITIES

DBE/NOVEMBER 2019

7.1 In the diagram, $A(-2;2)$, $B(-4;-6)$, $C(4;-2)$ and $D(6;y)$ are the vertices of a quadrilateral having AE//BC. D lies on BC such that $AD \perp BC$ and AC is drawn. $M(x;2)$ is a point on EC.



7.1.1 Calculate the gradient of BC (3) **L1**

7.1.2 If M is the midpoint of EC determine the values of x and y (3) **L2**

7.1.3 Calculate the length of BC (2) **L1**

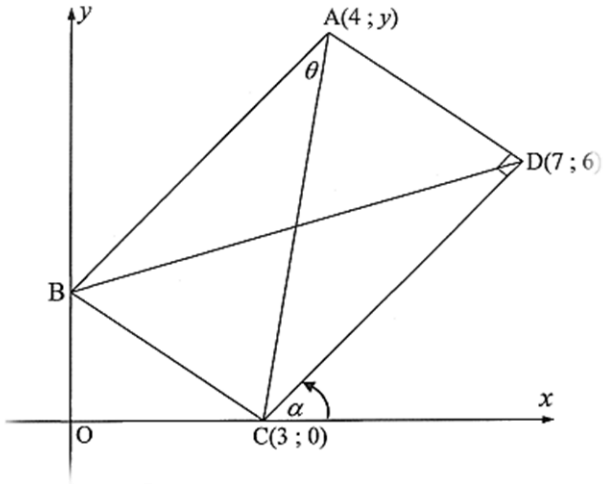
7.1.4 If it is further given that AE $=\sqrt{80}$, to which group of special quadrilaterals does ABCE belong? (1) **L2**

7.1.5 Determine the equation of AD in the form $y=mx+c$ (3) **L3**

7.1.6 Calculate the coordinates of D (5) **L3**

7.1.7 Determine the area ΔABC (3) **L2**

7.2 In the diagram, B is a point on the *y*-axis. A $(4;y)$, B, C $(3;0)$ and D $(7;6)$ are the vertices of a rectangle ADCB. Diagonals BD and AC are drawn. The inclination of DC is α and $B\hat{A}C = \theta$



7.2.1 Determine the gradient of CD (2) **L1**

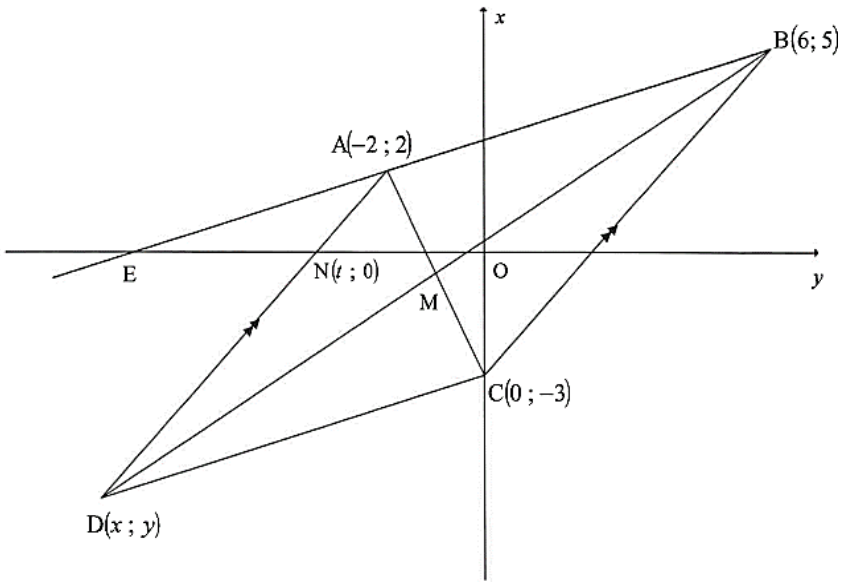
7.2.2 Calculate the size α (2) **L1**

7.2.3 Determine the value of y (4) **L3**

7.2.4 Calculate the size of θ (5) **L4**

DBE/NOVEMBER 2018

7.3 In the diagram, A $(-2;2)$, B $(6;5)$, C $(0;-3)$ and D $(x;y)$ are the vertices of a quadrilateral having AD//BC. BA produced has an x-intercept at E. BD and AC intersect at M. N $(t;0)$ is a point on A.



7.3.1 Calculate the gradient of BC (2) **L1**

7.3.2 Determine the equation of AD (3) **L2**

7.3.3 Determine value of t (2) **L2**

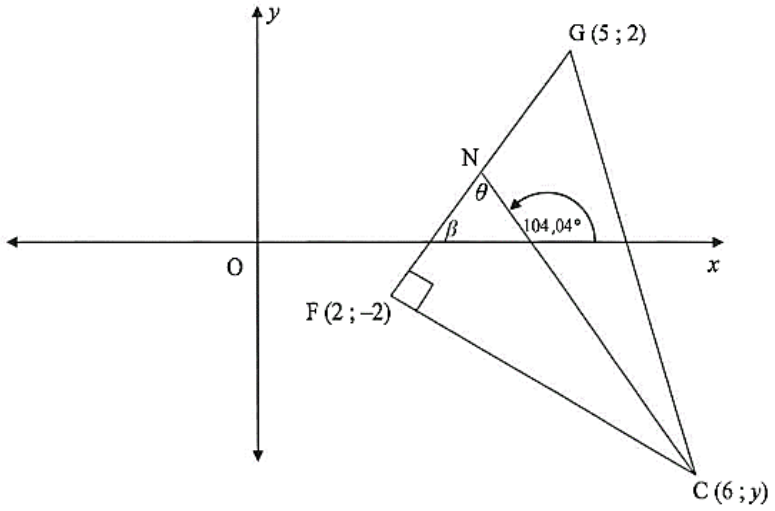
7.3.4 Calculate the length of AN (2) **L1**

7.3.5 If DC is defined by $y = \frac{3}{8}x - 3$, determine the coordinates of DC. (2) **L2**

7.3.6 Prove that ABCD is a parallelogram. (3) **L3**

7.3.7 Calculate the coordinates of M (3) **L2**

7.4 In the diagram $(2;-2)$, G $(5;2)$ and C$(6;y)$ are the vertices of ΔFGC. FG $\perp$ FC. N is a point on FG such that the inclination of NC is $104,04°$. The angle of inclination of FG is β and $F\hat{N}C = \theta$



7.4.1 Calculate the gradient of FG (2) **L1**

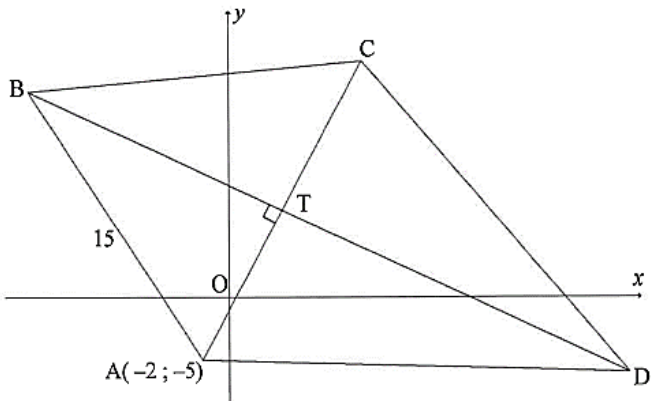
7.4.2 Calculate the value of y (3) **L3**

7.4.3 Calculate the size of θ (3) **L3**

7.4.4 Calculate the length of NC (4) **L4**

DBE/NOVEMBER 2017

7.5 A$(-2;-5)$, B, C, and D are the vertices of a quadrilateral ABCD such that diagonal AC is perpendicular to diagonal BD at T.
The equation of BTD is given by $2y + x = 18$ and AB = 15 units.



7.5.1 Determine the gradient of line AC (2) **L2**

7.5.2 Determine the equation of AC in the form $y = mx + c$ (2) **L2**

7.5.3 If the equation of AC is $y = 2x - 1$, calculate the coordinates of T (3) **L2**

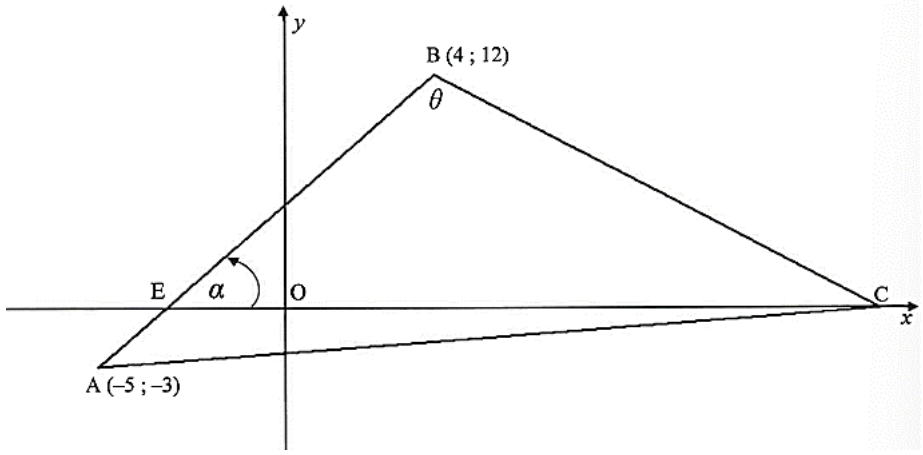
7.5.4 If ABCD is a kite with AB = BC

a) Determine the coordinates of C (2) **L**

b) Calculate the length of BT (4) **L3**

c) Calculate the perimeter of the circle passing through points B, C and T. (5) **L4**

7.6 C, a point on the x-axis, $A(-5;-3)$ and B $(4;12)$ are the vertices of a triangle. AB intersects the x-axis at E. $A\hat{B}C = \theta$ and $B\hat{E}C = \alpha$



7.6.1 Calculate the gradient of AB (2) **L1**

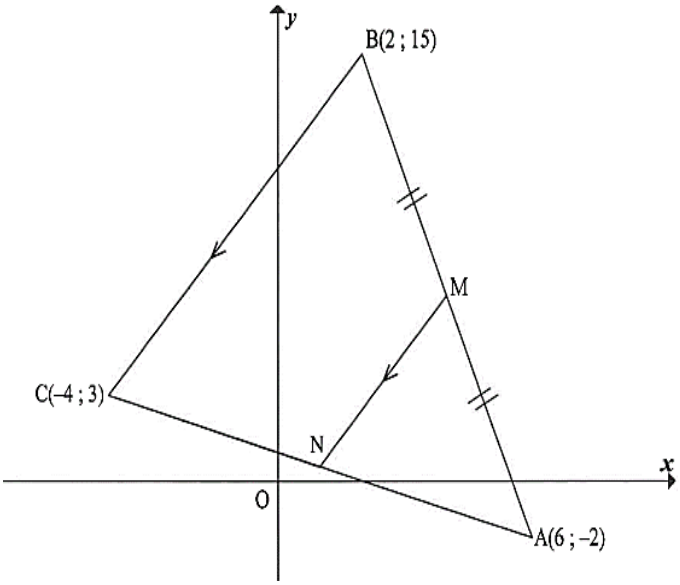
7.6.2 Determine the coordinates of point E (3) **L2**

7.6.3 Determine the size of α. Round off to the nearest whole number (2) **L1**

7.6.4 If $\theta = 76°$, determine the equation of the line through A parallel to BC (5) **L3**

DBE/NOVEMBER 2016

7.7 In the diagram, $A(6;-2)$, $B(12;15)$ and $C(-4;3)$ are the vertices of ΔABC. M is the midpoint of AB. N is a point on AC such that MN//BC.



7.7.1 Determine the coordinates of M, the midpoint of AB (2) **L1**

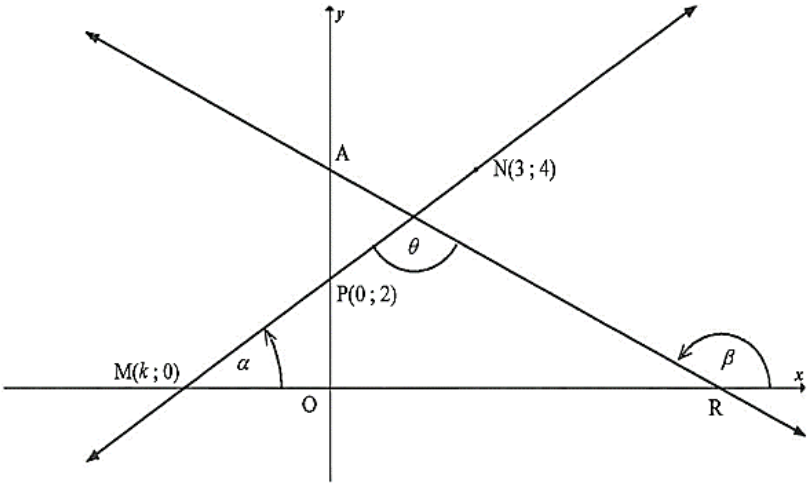
7.7.2 Determine the gradient of line MN (3) **L2**

7.7.3 Hence, or otherwise, determine the equation of line MN, in the form $y = mx + c$ (2) **L2**

7.7.4 Calculate, with reasons, the coordinates of point N (4) **L2**

7.7.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D (4) **L2**

7.8 In the diagram, R and A are the x- and y-intercepts respectively of the straight line AR. The equation of AR is $y=-\frac{1}{2}x+4$. Another straight line cuts the y-axis at P(0 ; 2) and passes through the points $M(k;0)$ and N(2 ; 4). α and β are the angles of inclination of the lines MN and AR respectively.



7.8.1 Given that M, P and N are collinear points, calculate the value of k. (3) **L2**

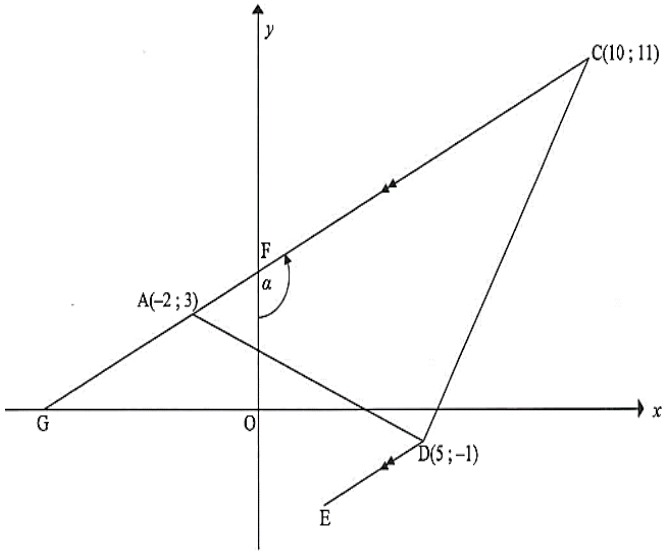
7.8.2 Determine the size of θ, the obtuse angle between the two lines (4) **L3**

7.8.3 Calculate the length of MR (3) **L3**

7.8.4 Calculate the area of ΔMNR (3) **L3**

DBE/NOVEMBER 2015

7.9 In the diagram, $A(-2;3)$, C(10 ; 11) and $D(5;-1)$ are the vertices of ΔACD. CA intersects the y-axis in F and CA produced cuts the x-axis in G. The straight line DE is drawn parallel to CA. $C\hat{F}O=\alpha$



7.9.1 Calculate the gradient of the line AC (2) **L1**

7.9.2 Determine the equation of the line DE in the form $y=mx+c$ (3) **L2**

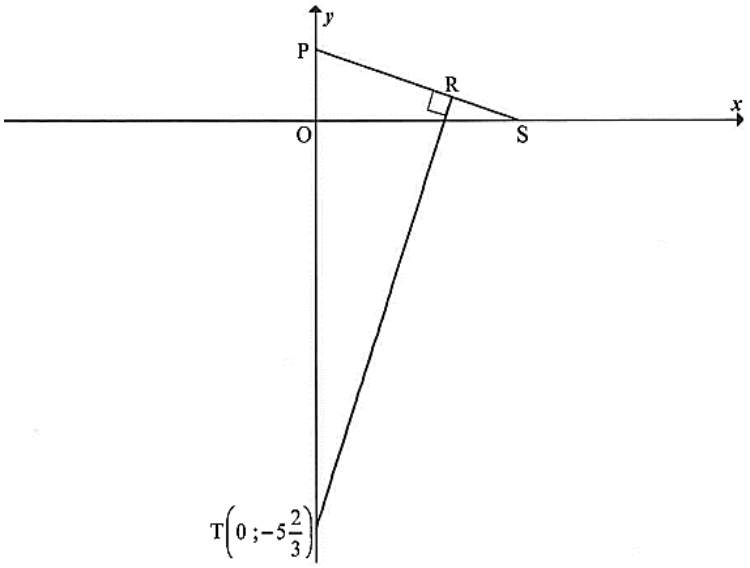
7.9.3 Calculate the size of α (3) **L3**

7.9.4 B is a point in the first quadrant such that ABDE, in that order, form a rectangle. Calculate, giving reasons, the

a) Coordinates of M, the midpoint of BE (3) **L3**

b) Length of diagonal BE (3) **L3**

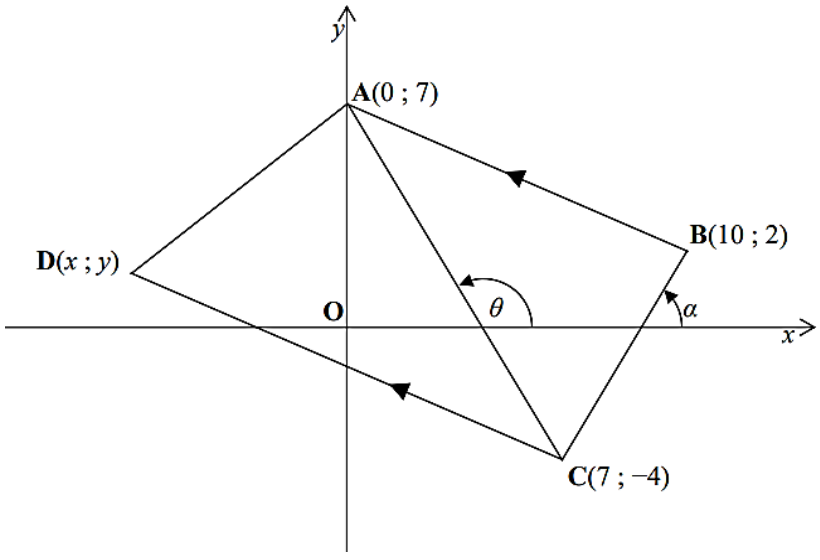
7.10 In the diagram, the straight line SP is drawn having S and P as its *x*- and *y*-intercepts respectively.
The equation of SP is $x+ay-a=0, a>0$.
It is also given that OS = 3OP.
The straight line RT is drawn with R on SP and RT $\perp$ PS.
RT cuts the *y*-axis in $T\left(0;5\frac{2}{3}\right)$.



7.10.1 Calculate the coordinates of P. (2) **L3**
7.10.2 Calculate the value of a (2) **L3**
7.10.3 Determine the equation of RT in the form $y=mx+c$ if it is given that $a=3$ (3) **L3**
7.10.4 Calculate the coordinates of R, the point where PS and TR meets. (4) **L2**
7.10.5 Calculate the area of ΔPRT if it is given that $R\left(2;\frac{1}{3}\right)$. (3) **L3**
7.10.6 Calculate, giving reasons, the radius of a circle passing through the points P, R and T. (2) **L4**

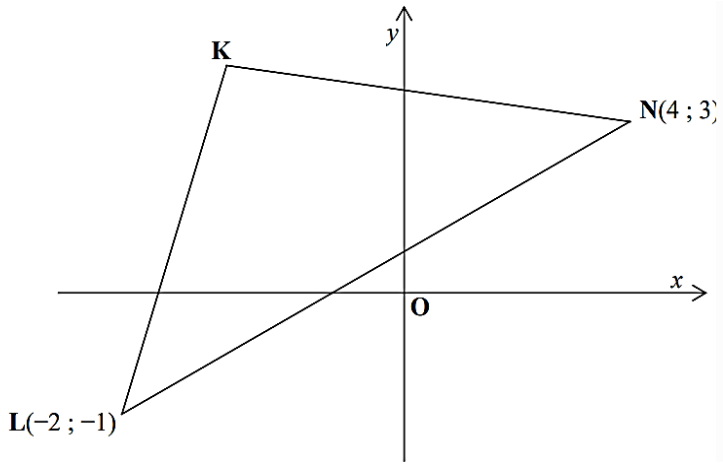
GAUTENG NOVEMBER 2022

7.11 In the diagram, the points A(0 ; 7), B(10 ; 2), C(7; −2) and D(*x*; *y*) form quadrilateral ABCD. AB//CD, with AC joined. The angles of inclination for AC are θ and α respectively.



Determine:
7.11.1 The length of AB (Leave your answer in simplified surd form) (3) **L1**
7.11.2 The size of α and θ (4) **L2**
7.11.3 The magnitude (size) of $B\hat{C}D$ (4) **L4**

7.12 Consider ΔKLN drawn below. KL has the equation $y = 5x + 9$ while KN has the equation $5y + x - 19 = 0$.



7.12.1 Show that the coordinates of K are $(-1;4)$. (3) **L2**

7.12.2 Show that KL $\perp$ KN. (3) **L2**

7.12.3 Hence, or otherwise, determine the area of ΔKNL. (4) **L3**

7.12.4 Determine the equation of the perpendicular bisector of LN. (4) **L3**

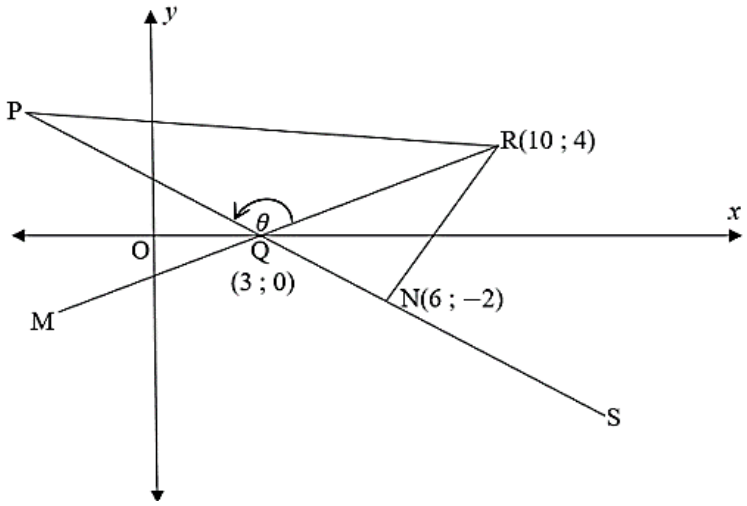
7.12.5 If L, N and P(7 ; *y*) are collinear, determine *y*. (2) **L2**

7.12.6 Determine the coordinates of Q, if KLQN is a parallelogram. (2) **L2**

7.12.7 Explain, with geometric reasons, why KLQN is a square. (2) **L3**

EC/NOVEMBER 2020

7.13 In the diagram below, straight line PS cuts the *x*-axis at Q(3 ; 0). MQR is a straight line which meets PR at R(10 ; 4). N$(6;-2)$ is a point on PS and RP is drawn. $P\hat{Q}R = \theta$



7.13.1 Determine the gradient of PS. (2) **L1**

7.13.2 Calculate the inclination of MR. (3) **L2**

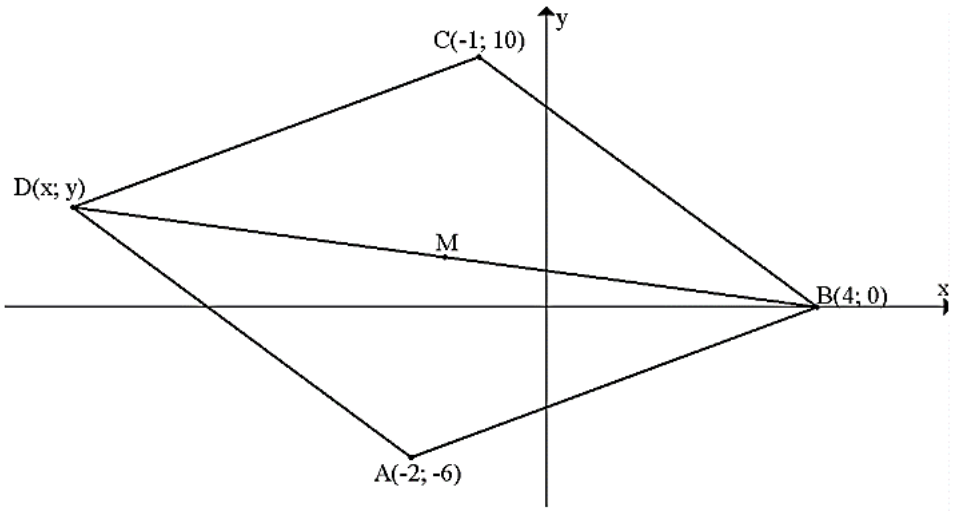
7.13.3 Determine the value of θ (3) **L3**

7.13.4 Prove that RN $\perp$ PS (3) **L2**

7.13.5 Calculate the area of ΔRQN (6) **L3**

7.13.6 Calculate the *y*-intercept of MR (4) **L2**

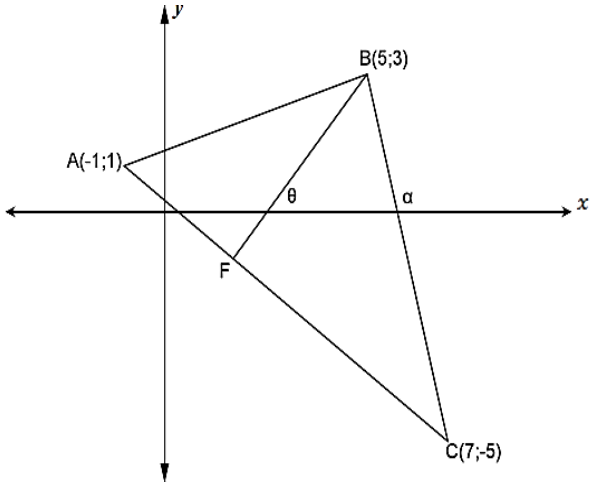
7.14 ABCD is a parallelogram with $A(-2;-6)$, B(4 ; 0), $C(-1;0)$ and $D(x;y)$ as shown below.



7.14.1 Calculate the length of BC (2) **L1**
7.14.2 Determine the gradient of AB (2) **L1**
7.14.3 Determine the equation of CD (3) **L2**
7.14.4 Determine the coordinates of M, the midpoint of BD (3) **L1**
7.14.5 HENCE, OR OTHERWISE, determine the values of x and y (3) **L2**

GAUTENG/JUNE 2018

7.15 In the sketch below the coordinates of the vertices of ΔABC are $A(-1;1)$, B(5 ; 3) and C(7 ; −5). Point F is a point on AC such that AF = CF. Line BF and line BC make angles θ and α respectively with the x-axis as indicated in the sketch.



7.15.1 Calculate the gradient of the line BC (2) **L1**
7.15.2 Calculate the coordinates of the point F (2) **L1**
7.15.3 Determine the equation of the median BF (3) **L2**
7.15.4 Calculate the size of $F\hat{B}C$. (rounded off to ONE decimal figure) (5) **L3**
7.15.5 If the coordinates of point K is (6 ; p), calculate the value of p if $A\hat{F}K = 90°$. (4) **L4**
7.15.6 Calculate the coordinates of point T, given that ABCT is a parallelogram. (2) **L2**
7.15.7 Prove that the diagonals of quadrilateral ABCT bisect each other (2) **L3**
7.15.8 Determine the perimeter of parallelogram LMNO which is an enlargement by a scale factor of TWO of parallelogram ABCT (5) **L4**

NW/JUNE 2018

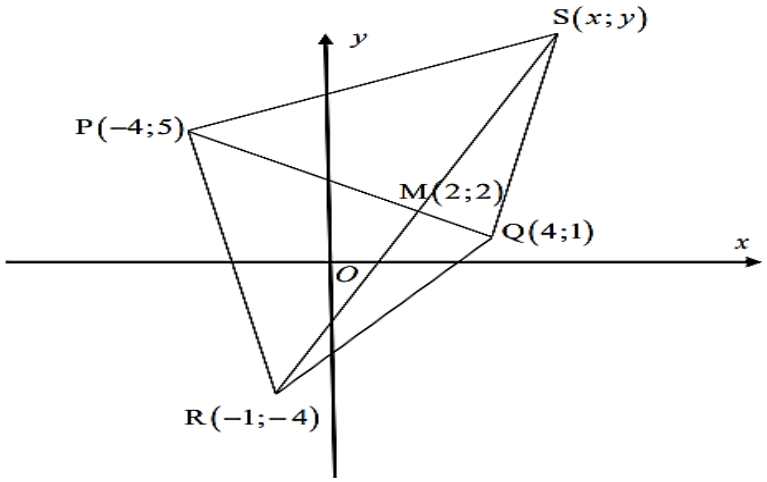
7.16 A $(-1;-1)$, B(2 ; 0) and C(5 ; p) are three points on the Cartesian plane.
Determine the value(s) of p if:

7.16.1 A, B, and C are collinear (2) **L2**

7.16.2 AB is perpendicular to BC (3) **L2**

7.16.3 The length of BC is 5 units (3) **L2**

7.17 In the diagram P(-4 ; 5), Q(4 ; 1) and R(-1 ; -4) are the vertices of a triangle in the Cartesian plane with M on PQ. M(2 ; 2) is a midpoint of straight line RS.



7.17.1 Determine the gradient of PQ (2) **L1**

7.17.2 Show that $P\widehat{M}S = 90°$ (3) **L2**

7.17.3 Determine the coordinates of S (3) **L2**

7.17.4 Prove that ΔQRS is isosceles (3) **L2**

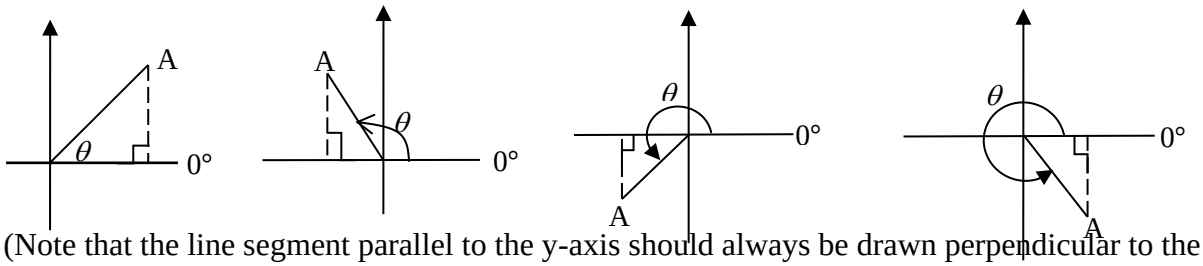
7.17.5 Determine the area of ΔPRS (5) **L3**

TOPIC	8. TRIGONOMETRY
	50 MARKS FROM PAPER 2
GUIDELINES, SUMMARY NOTES & STRATEGIES	
SUB-TOPIC	Use of diagrams to determine the numerical values of ratios for angles from 0° to 360°.
Concepts involved	• Definition of trig ratios • Interpretation of interval notation • Knowing in which quadrant a trig ratio is positive or negative (CAST RULE) • Application of Pythagoras theorem • Reduction formulae • Special angles • Identities

Approach to determine the numerical values of ratios for angles from 0° to 360°.

1. Write the given equation in a form of a simple trig ratio, for example,

   If $b\cos\theta - a = 0$ then $\cos\theta = \frac{a}{b}$; If $b\sin\theta - a = 0$ then $\sin\theta = \frac{a}{b}$; If $b\tan\theta - a$ then $\tan\theta = \frac{a}{b}$
2. Draw the sketch in the correct quadrant, using the given interval/restriction and where the trig ratio holds, for example



   (Note that the line segment parallel to the y-axis should always be drawn perpendicular to the *x*-axis)
3. Fill in the known details in the diagram drawn in the correct quadrant.
4. Use Pythagoras theorem to calculate the value of the unknown side. Decide whether the value is positive or negative. Remember the radius is always positive
5. Use the diagram to answer the questions asked based on the diagram
6. Do not use a calculator.

SUB-TOPIC	Trigonometric Identities

It must be noted that learners need to be able to derive and apply the fundamental identities.

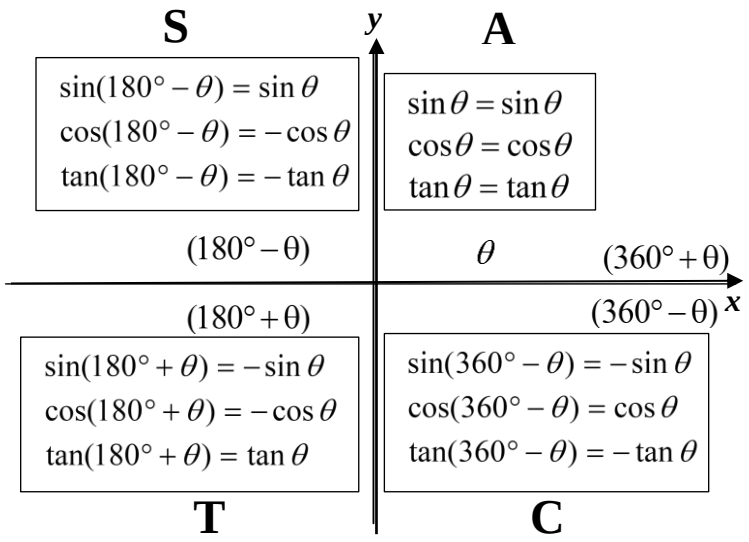
Two fundamental identities to be used:	**Also note:**	**From the square identity:**
(i) The quotient identity; $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (ii) Square identity: $\sin^2\theta + \cos^2\theta = 1$	$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\sin^2\theta = 1 - \cos^2\theta = (1-\cos\theta)(1+\cos\theta)$ $\cos^2\theta = 1 - \sin^2\theta = (1-\sin\theta)(1+\sin\theta)$

Tips on how to approach identities:

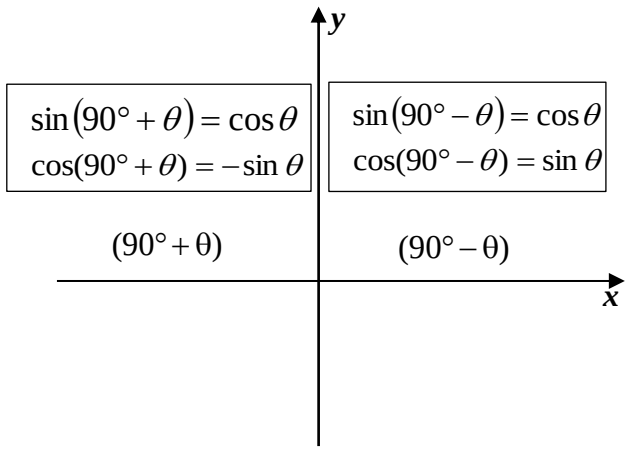
1. Choose the more complicated side (either LHS or RHS).
2. Express ***tanθ*** in terms of ***sinθ*** and ***cosθ***.
3. Simplify your chosen side **ALGEBRAICALLY** using the identities (in any of the forms), as far as possible or until it is the same/equal as/to the other side. (Sometimes it is necessary and sufficient to simplify both sides separately).
4. It is usually helpful to keep an eye on your required result (RHS/LHS) as it will give you the clue as to which identities would be the best to use.

SUB-TOPIC	Reduction formulae
Concepts involved	• Reduction of angles greater than $90°$ to acute angles using the following reduction formulae ($180° \pm ; 360° \pm$ and $90° \pm$) • For function values ($90° \pm$), each function changes to their co- function. • The **quadrant** should be identified and then the **sign** of the trig ratio in that quadrant should be noted. • Trigonometric identities. • Basic algebraic manipulations.

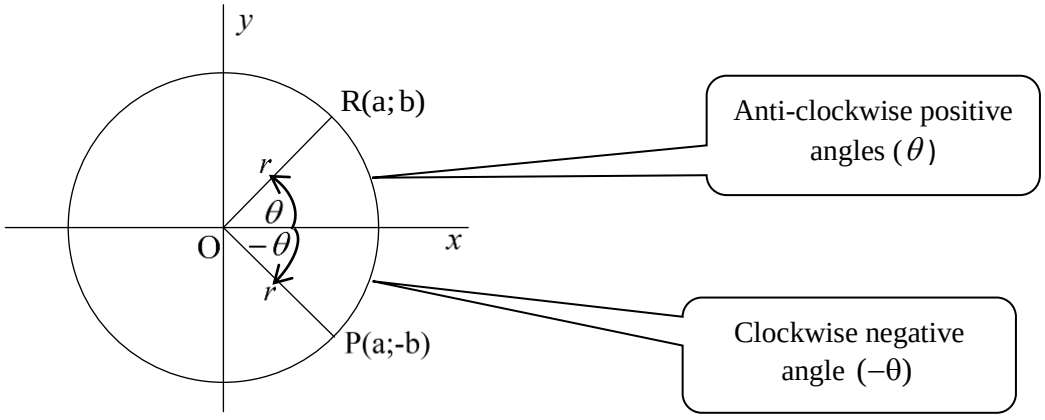
Reduction formulae for ($180° \pm$ and $360° \pm$)



Reduction formulae for (and $90° +$)



SUB-TOPIC	Negative Angles



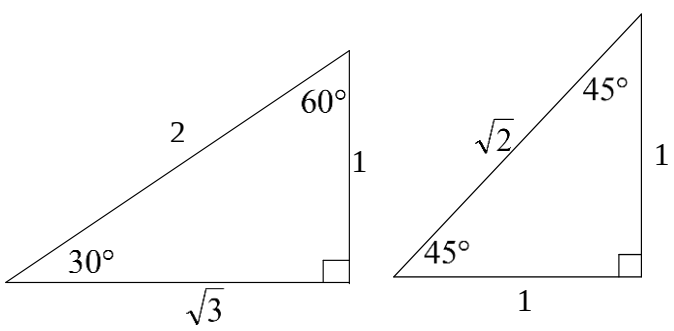
- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$

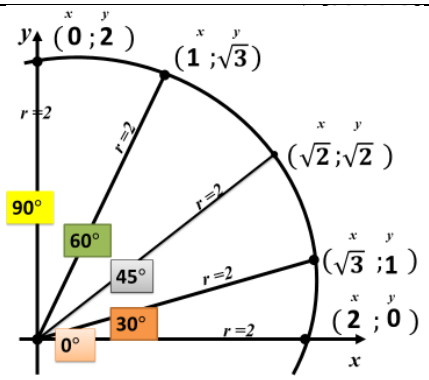
NB: Remember to change negative angles $(-\theta)$ to positive angles (θ)

SUB-TOPIC	Special Angles

- Be careful – Using a calculator is not allowed when dealing with special angles.

For example: Using a diagram: $\tan 30° = \frac{1}{\sqrt{3}}$ Using a calculator: $\tan 30° = \frac{\sqrt{3}}{3}$





F. SUBTOPIC	Solving Trig Equations And General Solution,

Hints for the solution of trigonometric equations:

- Simplify the equation by using identities where possible.
- Simplify the equation as much as possible until you have a trigonometric ratio of one angle equal to a constant/value.
- Factorization, as in the solution of algebraic equations, is often used. Look out for:
  - Common factor
  - Difference of two squares
  - Quadratic trinomials
  - Grouping of terms
- Take care not to divide by an unknown variable.
- Use the CAST rules to determine in which quadrants the angle will be.

SUBTOPIC	Solutions to triangles/2D problems

Strategies for solving 2 dimensional problems:

- Fill in all the given information on the diagram.
- Problems on 2D require you to solve a combinations of triangles in one plane
- Identify the right angled-triangle first (try to use Pythagoras or trig ratios in this triangle).
- Start to work in triangles with more information to find the common side.
- Label the side/angle as soon as you know its value.
- Use basic geometric results e.g. Exterior angle of a triangle, corresponding angle, co-interior angles, alternate angles, etc.

ACTIVITIES

8.1 **Use of diagrams to determine the numerical values of ratios for angles from 0° to 360°.**

8.1.1 If $3\tan A = 2$ and $90° < A < 360°$, determine the value of the following without the use of a calculator:

(a) $\sin A$ (3) **L2**

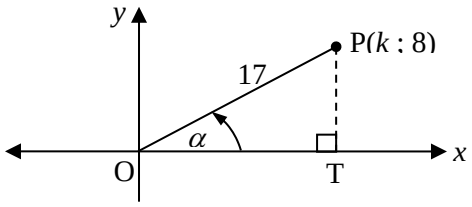
(b) $2\sin A\cos A$ (2) **L1**

8.1.2 If $\cos\theta = \frac{8}{17}$ and $\sin\theta < 0°$, find the value of:

(a) $\tan\theta$ (3) **L2**

(b) $\frac{\cos\theta}{\sin\theta}$ (2) **L2**

8.1.3 The point P(k ; 8) lies in the first quadrant such that OP = 17 units and $T\hat{O}P = \alpha$ as shown in the diagram alongside.



(a) Determine the value of k. (2) **L1**

(b) Write down the value of $\cos\alpha$. (1) **L1**

(c) If it is further given that $\alpha + \beta = 180°$, determine $\cos\beta$. (3) **L2**

(d) Hence, determine the value of $\sin\alpha\cos\beta - \cos\alpha\sin\beta$ (4) **L2**

8.1.4 Given: $\sin\alpha = \frac{3}{5}$ and $90^o < \alpha < 270^o$. With the aid of a sketch and **without using a calculator**, determine:

(a) $\tan\alpha$ (3) **L2**

(b) $\cos(90° + \alpha)$ (2) **L2**

8.1.5 If $\cos 27° = t$, express the following in terms of t without using a calculator.

(a) $\cos(-387°)$ (4) **L2**

(b) $\sin 333°$ (2) **L2**

(c) $\frac{\tan 153°}{\sin 207°}$ (3) **L2**

8.1.6 If $\cos\alpha = \sqrt{t}$, where α is an acute angle, express each of the following in terms of t:

(a) $\tan\alpha$ (3) **L2**

(b) $\sin(180° - \alpha)$ (2) **L1**

(c) $2\sin\alpha\cos\alpha$ (2) **L2**

8.1.7 If $3\sin\theta = 2$ and $\cos\theta < 0$,

(a) With an aid of a diagram determine the value of $3\tan^2\theta - 5\cos\theta$. (5) **L2**

(b) Calculate $\cos(180^o - \theta)$. (2) **L1**

(c) Determine the value of θ. (2) **L2**

8.1.8 If $\tan 20° = m$ determine the following in terms of ***m*** without the use of a calculator.

(a) $\sin 70°$ (3) **L2**

(b) $\tan^2 250°.\sin^2 560°$ (3) **L3**

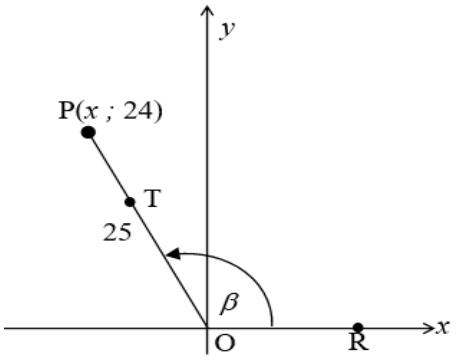
8.1.9 If $\tan\theta = \frac{3}{4}$, and $180^o < \theta < 270^o$, determine with an aid of a sketch the value of:

(a) $\sin\theta$ (3) **L2**

(b) $\cos(180° + \theta)$ (2) **L1**

(c) θ (answer to 2 decimal places). (2) **L2**

8.1.10 In the diagram alongside, P(x ; 24) is a point such that OP = 25 and $\hat{ROP} = \beta$, where β is an obtuse angle.



(a) Calculate the value of x. (2) **L2**

(b) Determine the value of each of the following WITHOUT using a calculator:

(i) $\sin\beta$ (1) **L1**

(ii) $\cos(180° - \beta)$ (2) **L2**

(iii) $\tan(-\beta)$ (2) **L2**

(c) T is a point on OP such that OT = 15. Determine the coordinates of T WITHOUT using a calculator. (3) **L3**

8.1.11 If $\sin 162° = p$, determine the following in terms of p

(a) $\sin 198°$ (3) **L3**

(b) $\cos(-108°)$ (2) **L3**

8.2 Trigonometric Identities.

8.2.1 Prove the following identities:

(a) $\dfrac{\sin\theta - \tan\theta.\cos^2\theta}{\cos\theta - 1 + \sin^2\theta} = \tan\theta$ (4) **L1**

(b) $\dfrac{\sin^3 x + \sin x.\cos^2 x}{\cos x} = \tan x$ (3) **L1**

(c) $\dfrac{\sin x - \sin x.\cos x}{\cos x - 1 + \sin^2 x} = \tan x$ (4) **L2**

(d) $1 - \dfrac{\cos^2 x}{1 + \sin x} = \sin x$ (4) **L2**

(e) $\dfrac{1}{\tan\alpha}\left(\sin\alpha.\tan\alpha + \cos\alpha\right) = \dfrac{1}{\sin\alpha}$ (4) **L2**

(f) $\dfrac{1}{\left(\cos x + 1\right)\left(\cos x - 1\right)} = \dfrac{-1}{\tan^2 x.\cos^2 x}$ (4) **L2**

(g) $\dfrac{1 - \tan\theta}{1 + \tan\theta} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$ (4) **L2**

(h) $\left(\tan y + \dfrac{1}{\tan y}\right)\left(1 - \cos^2 y\right) = \tan y$ (6) **L3**

(h) $\dfrac{2\sin^2 x}{2\tan x - 2\sin x\cos x} = \dfrac{\cos x}{\sin x}$ (4) **L3**

(i) $\dfrac{\sin^2 x - \cos^2 x}{\cos x\left[\sin\left(180° - x\right) - \cos x\right]} - 1 = \tan x$ (4) **L4**

Reduction formulae and special angles

8.2.2 Simplify the following

(a) $\sqrt{\dfrac{\sin(-390°)}{\cos 240°} + \tan(180° + x)\cos(180° + x).\cos(90° - x)}$ (8) **L3**

(b) $\dfrac{\sin(-338°).\cos 300°}{\cos 248°.\tan 135°}$ (6) **L2**

(c) $\dfrac{\sin 120°.\cos 210°.\tan 315°.\cos 27°}{\sin 63°.\cos 540°}$ (7) **L2**

(d) $\dfrac{\cos(-33°).\tan 147°}{2\cos 303°.\sin 240°}$ (7) **L3**

(e) $\dfrac{\sin 210°.\cos 790°.\tan(-330°)}{\sin 160°}$ (5) **L2**

(f) $\cos^2 253° + \cos^2 557°$ (4) **L2**

(g) $\dfrac{\cos(720° - x).\sin(360° + x).\tan(x - 180°)}{\sin(-x).\cos(90° - x)}$ (6) **L2**

(h) $\dfrac{\sin(90° + x).\tan(360° + x)}{\sin(180° + x).\cos(90° - x) + \cos(540° + x).\cos(-x)}$ (7) **L3**

(i) $\dfrac{\cos(90°+A)\sin(180°+A)}{\tan 225° - \cos^2(-A)}$ (6) **L2**

(j) $\dfrac{\cos(-x).\tan 225°}{\sin 90° - \sin x} + \tan(360° - x)$ (7) **L4**

(k) $\sqrt{\dfrac{1-\tan^2\theta.\cos(-\theta).\cos(360^o-\theta).\tan 225^o}{\left[\sin 90^o - \sin(180^o+\theta)\right]\left[\sin 90^o - \cos(90°-\theta)\right]}}$ (6) **L3**

8.3 Trigonometric equations and general solution

8.3.1 Determine the general solution:

(a) $2\cos^2\theta - \cos\theta = 0$ (6) **L2**

(b) $2\sin x\cos x - \cos^2 x = 0$ (6) **L3**

(c) $\sqrt{3}\cos\theta + \sin\theta = 0$ (6) **L2**

(d) $2\sin^2\theta - \sin\theta = 1$ (6) **L2**

(e) $\sin(2\theta + 30°) = -0,74$ (4) **L2**

(f) $\cos(x + 30°) = \sin 2x$ (6) **L3**

(g) $3\sin^2 x + 2\sin x\cos x = 4\cos x + 6\sin x$ (5) **L3**

(h) $10\sin x\cos x + 5\cos x - 6\sin x - 3 = 0$ (6) **L3**

Solve for *x:*

(g) $3\cos^2 x + 10\sin x + 5 = 0$; where $x \in [0°; 360°]$ (6) **L3**

(h) $3\tan x - \dfrac{5}{\tan x} = 2$; where $x \in [-180°; 180°]$ (6) **L2**

(i) $\sin 2x = 4\cos 2x$; where $x \in [0°; 270°]$ (5) **L2**

(j) $\cos(x - 25°) = -\sin 2x$;where $x \in [-180°; 180°]$ (6) **L3**

(k) Given that: $\left(\tan y + \dfrac{1}{\tan y}\right)\left(1 - \cos^2 y\right) = \tan y$

For which values of *y* will the identity be undefined where $y \in [0°; 180°]$ (3) **L3**

(l) Given that: $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{2}{\cos x}$

For which values of *x* will the identity be undefined where $x \in [90°; 270°]$ (2) **L2**

8.4.1 (a) On the same set of axes, sketch the graphs of $f(x)=\sin(x-60^\circ)$ and $g(x)=\cos 2x$ for the interval $-180^\circ \le x \le 180^\circ$. Indicate clearly the intercepts with the axes and the turning points. (6) **L2**

(b) Write down the range of *f*. (1) **L1**

(c) Write down the period of *g*. (1) **L1**

(d) Determine the general solution of $\sin(x-60^\circ)=\cos 2x$. (5) **L3**

(e) Hence, for which values of x in the interval $-180^\circ \le x \le 0^\circ$ will $f(x) \ge g(x)$? (3) **L2**

(f) If $p(x)=-\cos(2x+20^\circ)$, describe fully, in words, the transformation from *g* to *p*. (3) **L3**

8.4.2 Consider the functions defined by $f(x)=\sin 2x$ and $g(x)=\cos(x+60^\circ)$.

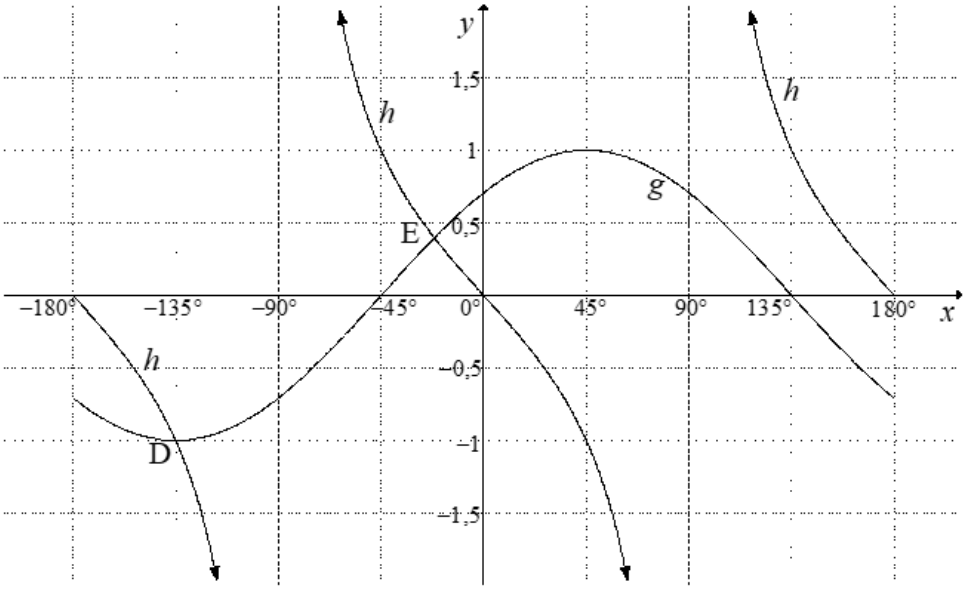
(a) Draw a neat sketch of the curves of ***f*** and ***g*** for $-180^\circ \le \theta \le 180^\circ$, clearly indicating the intercepts with the axes. (6) **L2**

(b) Write down the range for *g*. (1) **L1**

(c) Write down the period of *f*. (1) **L1**

(d) For which value(s) of ***x*** is the graph of ***g*** decreasing? (2) **L2**

8.4.3 In the diagram below the graphs of $g(x)=\sin(x+a)$ and $h(x)=b\tan cx$ are drawn in the interval $x \in [-180^\circ ; 180^\circ]$. *g* and *h* intersect at $D(-135^\circ ; -1)$ and $E(-22,3^\circ ; 0,4)$.



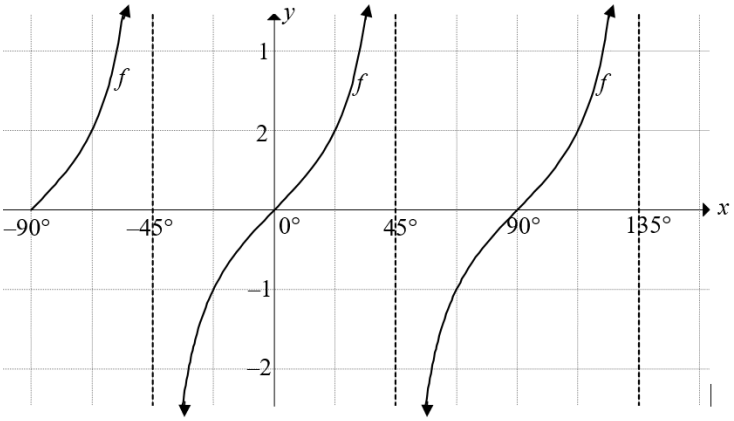
(a) Write down the values of *a*, *b* and *c*. (3) **L1**

(b) Write down the equations of the asymptotes of *h* in the interval $[-180^\circ ; 180^\circ]$. (2) **L1**

(c) Solve for x if $g(x)-h(x) \ge 0$ in the interval $[-180^\circ ; 0^\circ]$. (4) **L3**

(d) Describe the transformation of *g* to *m* if $m(x)=\cos x-1$. (2) **L4**

8.4.4 In the diagram, the graph of $f(x) = \tan bx$ is drawn in the interval $-90° \le x \le 135°$

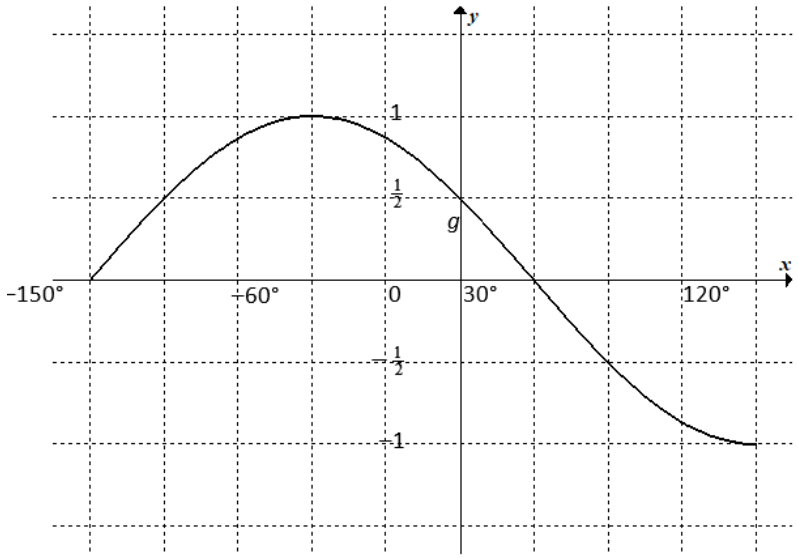


(a) Determine the value of *b*. (1) **L1**

(b) Determine the values of *x* in the interval $0° \le x \le 135°$ for which $f(x) \le -1$. (2) **L3**

(c) Graph *h* is defined as $h(x) = \tan b(x + 55°)$. Write down the equations of the asymptotes of *h* in the interval $-90° \le x \le 135°$. (2) **L2**

8.4.5 In the diagram, the graph of $g(x) = \cos(x + 60°)$ is drawn for the interval $-150° \le x \le 120°$.



(a) On the same system of axes, draw the graph of $k(x) = -\sin x$ for the interval $-150° \le x \le 120°$. Show all the intercepts with the axes as well as the coordinates of the turning points and end points of the graph. (4) **L2**

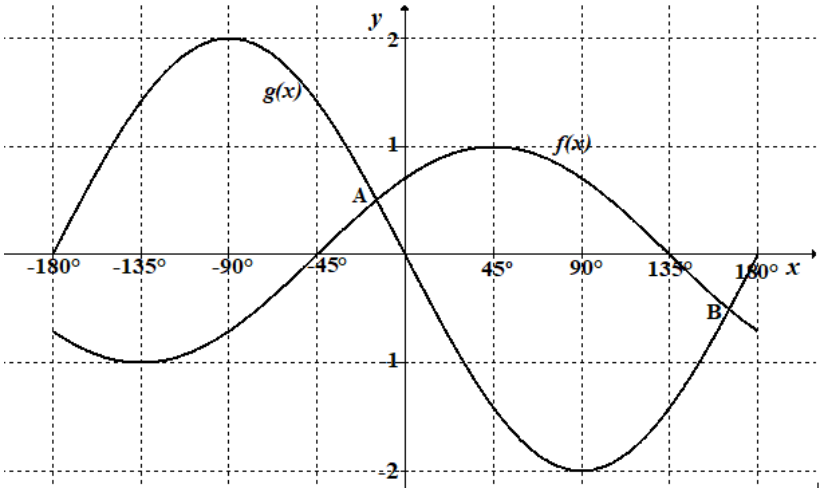
(b) Determine the minimum value of $h(x) = \cos(x + 60°) - 3$. (2) **L2**

(c) Solve the equation $\cos(x + 60°) + \sin x = 0$ for the interval $-150° \le x \le 120°$. (6) **L3**

(d) Determine the values of *x* for the interval $-150° \le x \le 120°$ for which $\cos(x + 60°) + \sin x > 0$. (2) **L3**

(e) The function *g* can also be defined as $y = -\sin(x - \theta)$, where θ is an acute angle. Determine the value of θ. (2) **L4**

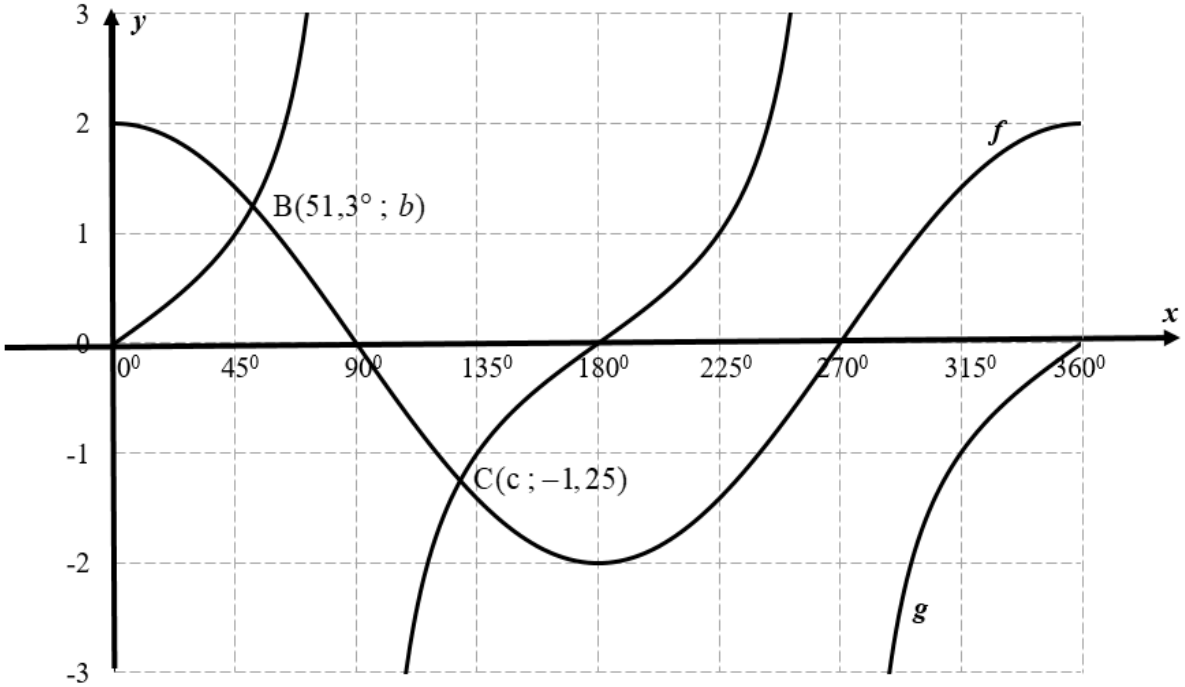
8.4.6 The following sketch represents the graphs of $f(x)=a\cos(x+b)$ and $g(x)=c\sin x$ for $x\in[-180°;180°]$.



(a) From the sketch above deduce the values of a, b and c. (3) **L1**

(b) $A(-14,\ 64^o;p)$ and $B(q;\ -0,51)$ are the coordinates of the points of intersection of f and g. Write down, rounded off to 2 decimal places, the values of p and q. (2) **L2**

8.4.7 The figure shows sketch graphs of $f(x)=a\cos x$ and $g(x)=\tan x$ for the interval $0°\le x\le 360°$



(a) Write down the value of a. (1) **L1**

(b) Determine:

(i) The amplitude of f. (1) **L1**

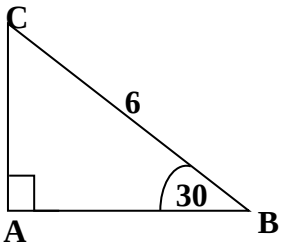
(ii) The period of g. (1) **L1**

(c) If the graphs of f and g intersect at $B(51,3° ; b)$ and $C(c ; -1,25)$ write down the values of b and c without the use of a calculator. (2) **L1**

(d) Use the graphs to write down **ONE** value of x for which $g(x)-f(x)=2$ (1) **L2**

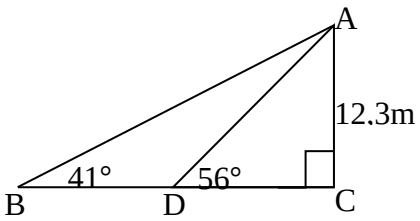
8.5 Problems in 2-Dimensions

8.5.1 Use the diagram alongside to calculate the length of AB and AC



(2) **L1**

8.5.2 In ΔABC, AC ⊥ BC, AB =12,3m, $\hat{B} = 41°$ *and* $A\hat{D}C = 56°$

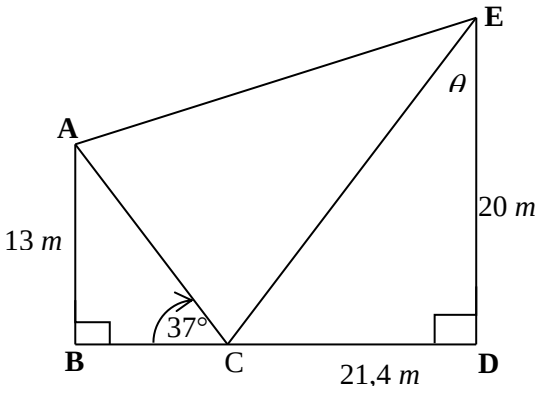


(a) Calculate the length of DC (2) **L1**

(b) Calculate the length of BC (2) **L1**

(c) Hence calculate the length of BD (1) **L1**

8.5.3 AB and DE are two Towers in the same horizontal plane.
From point C the angle of elevation of A, the top of tower AB, is 37°.
AB is 13*m* high, DE = 20*m* high and CD = 21,4*m*.



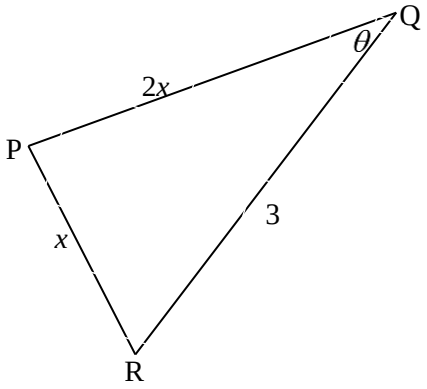
Calculate:

(a) the length of AC (2) **L1**

(b) the value of θ (2) **L1**

(c) the area of ΔECD (2) **L1**

8.4 In Δ PQR, QR = 3 units, PR = *x* units, PQ = 2*x* units and $P\hat{Q}R = \theta$ **(Gr 11 P2 Nov 2017;DBE)**

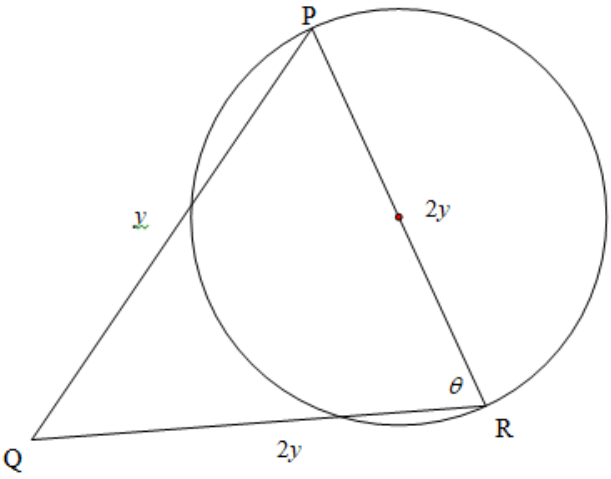


(a) Show that $\cos\theta = \dfrac{x^2+3}{4x}$ (4) **L2**

(b) Hence, calculate the area of Δ PQR , if *x* = 2,4 units: (4) **L3**

(c) Calculate the values of *x* for which the triangle exists. (4) **L4**

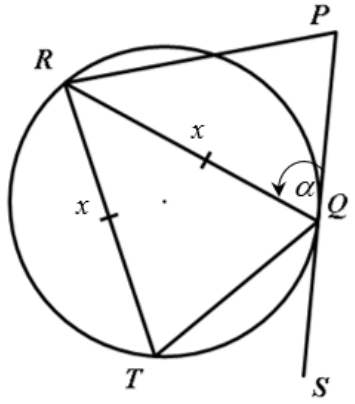
8.5.5 In the diagram, PR is a diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\hat{R} = \theta$, PR = QR = $2y$ and PQ = y.



(a) Determine the value of $\cos\theta$. (4) **L3**

(b) If QR cuts the circumference of the circle at T, determine PT in terms of y and θ. (3) **L4**

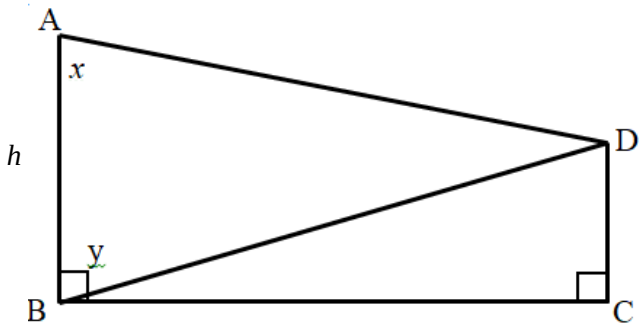
8.5.6 In the figure below RT = RQ = x, $P\hat{Q}R = \alpha$ and PQS is a tangent to the circle.



(a) Prove that $TQ = x\sqrt{2(1+\cos 2\alpha)}$ (4) **L4**

(b) If $\alpha = 63^\circ$ and $x = 17\,\text{cm}$, calculate the area of ΔRQT. (3) **L3**

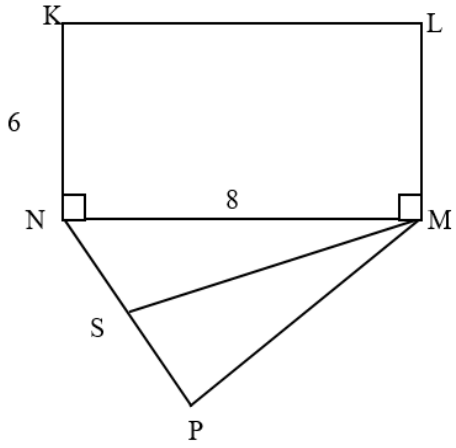
8.5.7 In the diagram below, $B\hat{A}D = x$, $A\hat{B}D = y$ and $AB = h$, AB and DC are perpendicular to BC.



(a) Determine the distance of BD in terms of x, y and h (3) **L2**

(b) Hence show that $DC = \dfrac{h\sin x\cos y}{\sin(x+y)}$ (3) **L3**

8.5.8 The figure consists of a rectangle KLMN and ΔNMP. The area of pentagon KLMPN is 63 square units.
KN = 6 units, NM = 8 units and NP = 7 units.



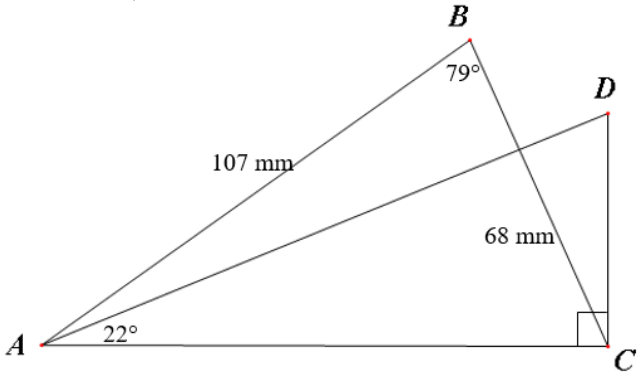
(a) Determine the area of Δ NMP (2) **L2**

(b) Calculate the size of acute $M\hat{N}P$. (3) **L2**

(c) If MS is drawn to meet NP at S and SM = 5,2 units, calculate $N\hat{S}M$. (3) **L2**

8.5.9 Given ΔPQR, with PQ = 140 mm, QR = 123,4 mm and PR = 199,2 mm.
Calculate the size of the largest angle of ΔPQR. (4) **L3**

8.5.10 In the figure below, AB = 107 mm, BC = 68 mm and $A\hat{B}C = 79^o$. AC ⊥ DC. $D\hat{A}C = 22^o$.

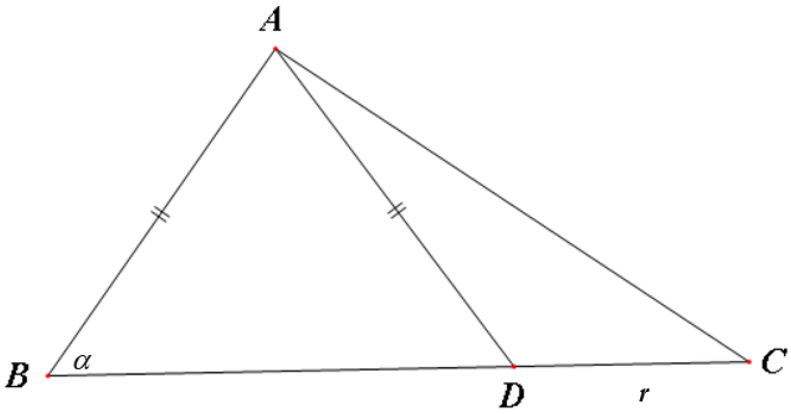


(a) Calculate the length of AC. (3) **L2**

(b) Calculate the length of DC. (2) **L2**

8.5.11 Complete: Area of Δ ABC = $\frac{1}{2}$ ab …….. (1) **L1**

8.5.12 In the figure below AB = AD with DC = $\frac{1}{2}$BD and DC = *r*. It is also given that $A\hat{B}D = \alpha$ and $\sin 2\alpha = 2\sin\alpha \cos\alpha$.



(a) Show that AD = $\frac{r}{\cos\alpha}$ (4) **L4**

(b) Now determine the area of Δ ADC, in terms of α and *r*. (4) **L2**

TOPIC	9. EUCLIDEAN GEOMETRY

GUIDELINES, SUMMARY NOTES, & STRATEGIES

1. WAYS IN WHICH EUCLIDEAN GEOMETRY IS TESTED

1. Completing a statement of a theorem in words.
2. Determining the value of an angle in **two ways:** numerical and/or in terms of the variable(s)
3. **Proofs in riders:** Direct and indirect proofs
4. **Examinable proofs** to be known
   4.1 Line from the centre ⊥ chord
   4.2 Line from centre to midpoint of chord
   4.3 Angle at the centre is twice the angle at the circumference
   4.4 Opposite angles of a cyclic quad are supplementary
   4.5 Tan chord theorem

2. COMPLETING A STATEMENT OF A THEOREM IN WORDS.

- Know by heart all the theorems and be able to complete the statement.

3. DETERMINING THE VALUE OF AN ANGLE

- Know all the theorems about **lines**, **triangles** and **circles (Centre** group, **non-centre** group, **tangent** group and **cyclic quad** group**)**.
- Every statement must come with a reason and reasons must be stated according to the list of acceptable reasons from the exam guidelines
  e.g. for isosceles triangles the acceptable reason is: ∠s opp = sides

4. PROOFS IN RIDERS

Know how theorems and their converses are being formed in diagrams.

- When given 3 points on the circumference look out for a possibility of a triangle. If one side is produced then you may expect exterior angle of a triangle. If there is a tangent on the circle, then there is a possibility of having a Tan Chord Theorem
- When given 4 or 5 points on the circumference then there is a possibility that 4 points may be joined and then there is a cyclic quad. In a case that one side is produced then you may expect exterior angles of a cyclic quad.
- Start with a given angle linking with what is required to prove
- Visualization: Mind picture of diagrams of theorems

5. DIRECT AND INDIRECT PROOFS IN RIDERS.

- In Geometry we mostly use angles to prove questions.

1. Direct proof question: Prove $\hat{A} = \hat{B}$

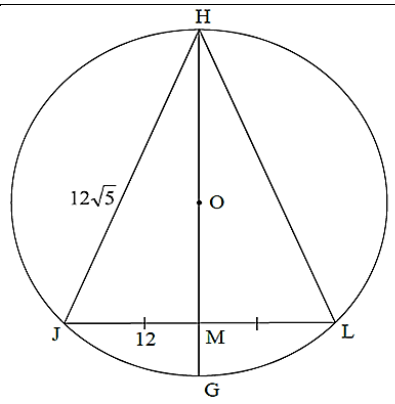
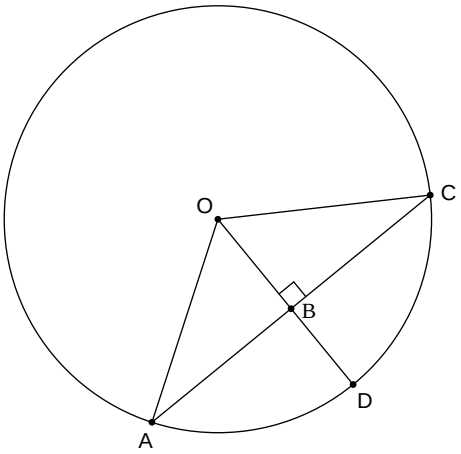
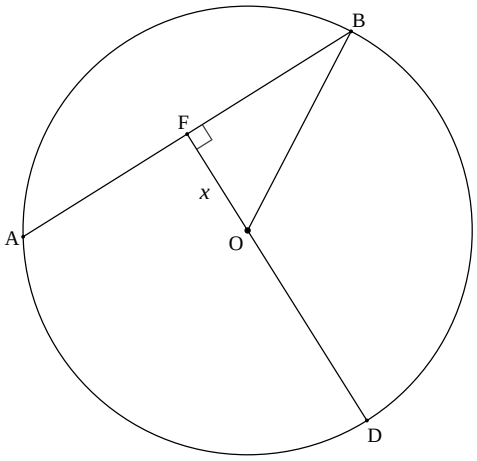
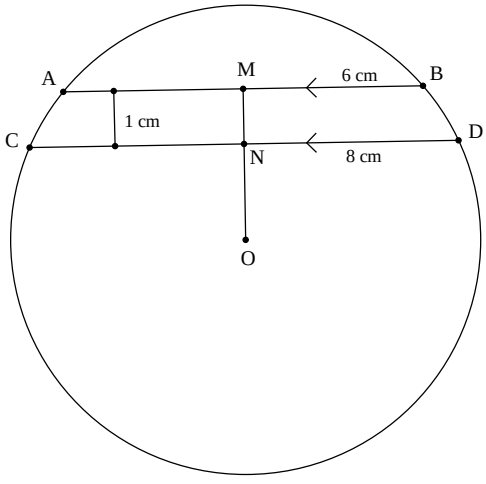
2. Indirect proof question: Prove that a line is parallel to another line.

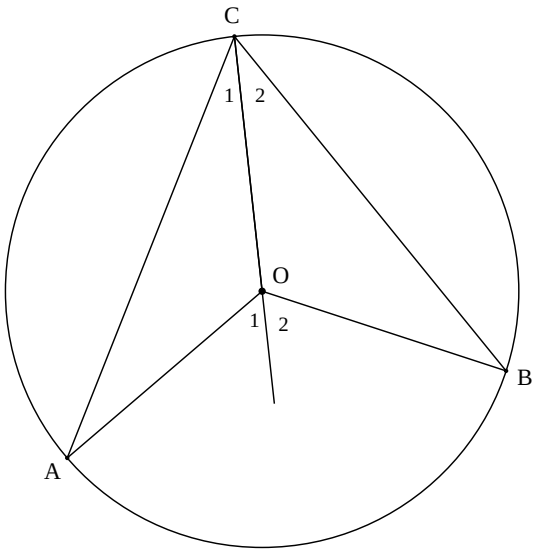
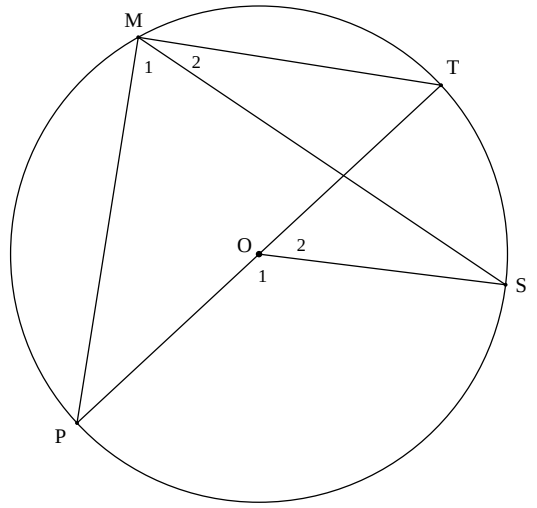
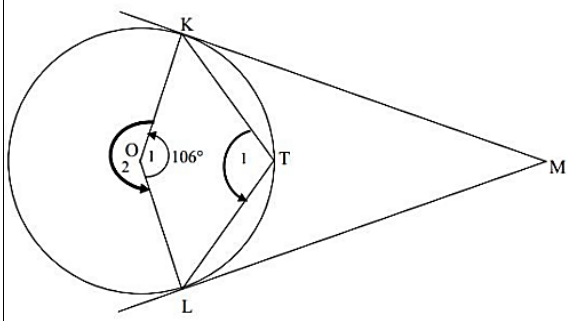
Remember in Euclidean geometry- we mostly use angles to prove. This question is not asking about the angles directly. Here we need to prove sides but using angles i**ndirectly. Why indirectly?** Because we mostly use angles to prove.

∴ First, we need to change this question to be direct, and then prove. If we say it must be direct we mean that it must ask to prove angles 1st, then conclude by stating the sides that are parallel

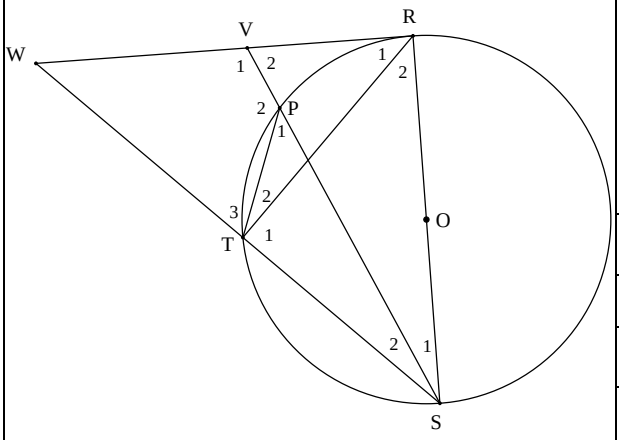
NB!!!!!

- Do not make any assumption e.g. do not assume that a line is a tangent or a diameter, unless you are told that it is.
- Look for key words in the statement such as **centre, parallel lines, tangents, cyclic quadrilaterals, bisects, etc**.
- Continuously update the diagram as you read the statement and as you find the angles.
- When proving theorems, **no construction, no marks.**
- You will not always be told that you have a cyclic quadrilateral. Therefore, check lines joining four points on the circumference.
- For every statement there **must** be a reason.

	ACTIVITIES			
9.1		In the diagram along, a circle centred at O is drawn. H, J, G and L are points on the circle. ΔHJL is drawn. HOG bisects JL at M. HJ $=12\sqrt{5}$ units and JM$=$12 units.		
		9.1.1 If MG $= 6$ units and OM $= x$, write HM in terms of x.	(2)	**L2**
		9.1.2 Calculate, giving reasons, the length of the radius of the circle	(5)	**L3**
			[7]	
9.2		In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. OB $= 2x$ units and AC $= 8x$ units. Show that the length of BD is $2\left(\sqrt{5}-1\right)$.	(5)	**L3**
9.3		In the diagram, O is the centre ABD. F is a point on chord AB such that DOF $\perp$ AB. AB $=$ FD $= 8$cm and OF $= x$ cm. Determine the length of the radius of the circle.	(5)	**L3**
9.4		AB and CD are two parallel chords on the same side of the centre O of a circle. The shortest distance between AB and CD is 1 cm . The length of AB is 6 cm and of CD is 8 cm. M is the midpoint of AB and the line OM intersects CD at N which is the midpoint of CD. Calculate the length of the radius of the circle	(4)	**L4**

9.5		In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chord AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$			
		9.5.1	Determine the size of $\hat{O}_1$ in terms of x.	(3)	**L2**
		9.5.2	Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $A\hat{O}B = 2A\hat{C}B$.	(3)	**L2**
				[6]	
9.6		In the diagram, PT is a diameter of the circle centre O. M and S are points on the circle on either side of PT. MP, MT, MS and OS are drawn. $\hat{M}_2 = 37°$ Calculate, with reasons, the size of:			
		9.6.1	$\hat{M}_1$	(2)	**L2**
		9.6.2	$\hat{O}_1$	(2)	**L1**
				[4]	
9.7		In the diagram, O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and TL are joined. $\hat{O}_1 = 106°$			
		9.7.1	Calculate, with reasons, the size of $\hat{T}_1$.	(3)	**L2**
		9.7.2	Prove that the quadrilateral OKML is a kite.	(3)	**L2**
		9.7.3	Prove that quadrilateral OKML is a cyclic quadrilateral.	(3)	**L2**
		9.7.4	Calculate, with reasons, the size of $\hat{M}$.	(2)	**L2**
				[11]	

9.8

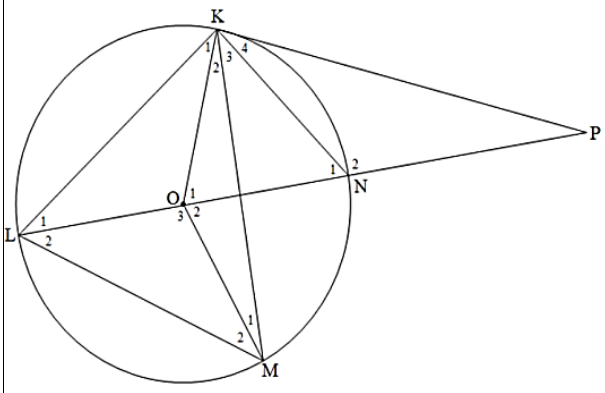


In the diagram below, RS is a diameter of the circle centred at O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\hat{R}_1 = 50°$

Calculate the sizes of the following angles.

9.8.1	$\hat{R}_2$	(3)	**L2**
9.8.2	$\hat{W}$	(3)	**L3**
9.8.3	$\hat{P}_1$	(2)	**L2**
9.8.4	Prove that $\hat{V}_1 = P\hat{T}S$	(4)	**L4**
9.8.5	Hence, prove that WVPT is a cyclic quadrilateral.	(1)	**L1**
		[13]	

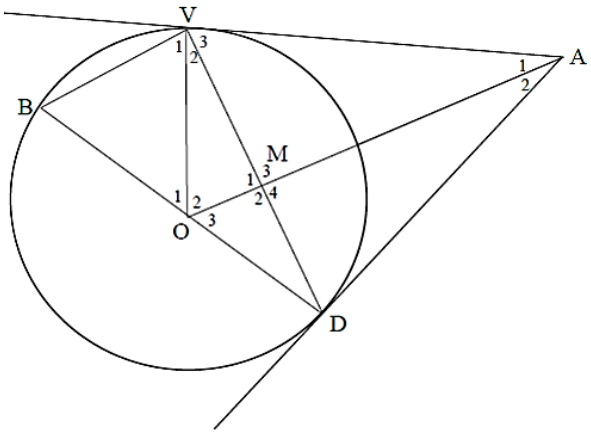
9.9



In the diagram, O is the centre of the circle and KP is the tangent to the circle. LN, the diameter of the circle, is extended to meet KP at P. Straight lines OK, OM, KM and KN are drawn.

9.9.1	Write down two angle equals to 90°	(2)	**L1**
9.9.2	If $\hat{K}_4 = x$, write down the following angles in terms of x, giving reasons.		
	(a) $\hat{L}_1$	(2)	**L1**
	(b) $\hat{K}_1$	(2)	**L1**
	(c) $\hat{O}_1$	(2)	**L2**
9.9.3	Join MP, which is a tangent to the circle, and hence prove that KOMP is a cyclic quadrilateral.	(3)	**L3**
		[11]	

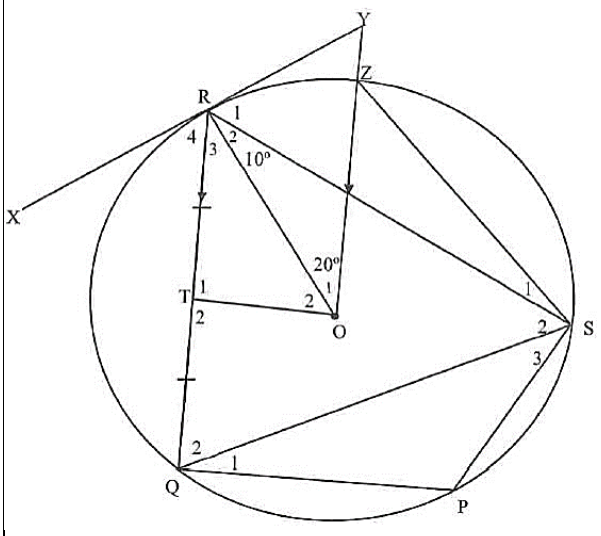
9.10



From a point A outside the circle, centre O, two tangents AD and AV are drawn. AO and VD intersect at M. BOD is a diameter of the circle. BV and VO are drawn, $V\hat{A}D = 40°$.

9.10.1	Prove that quadrilateral VODA is cyclic.	(2)	**L2**
9.10.2	Calculate the magnitude of $\hat{O}_1$	(2)	**L3**
9.10.3	Prove that $BV \parallel OA$.	(5)	**L3**
		[9]	

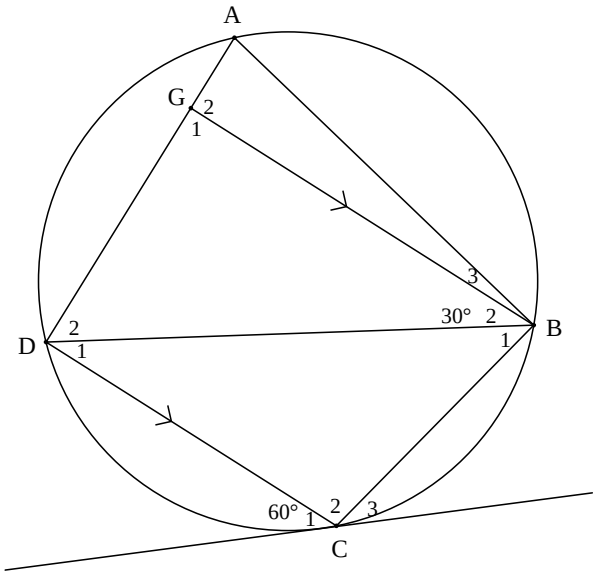
9.11



In the diagram below, points P,Q,R and S are points on a circle with centre O. OT bisects QR at T. XRY is a tangent to the circle at point R. OZ is produced to meet at Y where $OY \parallel QR$. $R\hat{O}Y = 20°$ and $S\hat{R}O = 10°$. Chord SZ is drawn.

9.11.1	Calculate, with reasons, the size of the following angles,			
	(a)	$\hat{S}_1$	(2)	**L1**
	(b)	$\hat{R}_3$	(1)	**L1**
	(c)	$\hat{P}$	(2)	**L2**
	(d)	$\hat{S}_2$	(4)	**L3**
9.11.2	Prove that XRY is a tangent to the circle passing through R, T and O.		(3)	**L3**
			[12]	

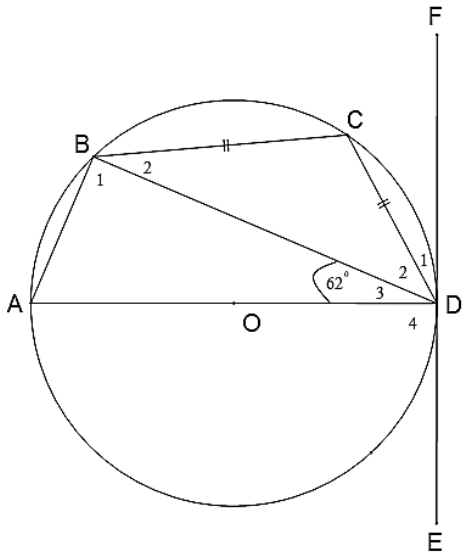
9.12



In the diagram, ABCD is a cyclic quadrilateral. G is a point on AD such that $BG \parallel CD$. ECF is a tangent to the circle at C. BD is a chord to the circle. $G\hat{B}D = 30°$ and $D\hat{C}E = 60°$

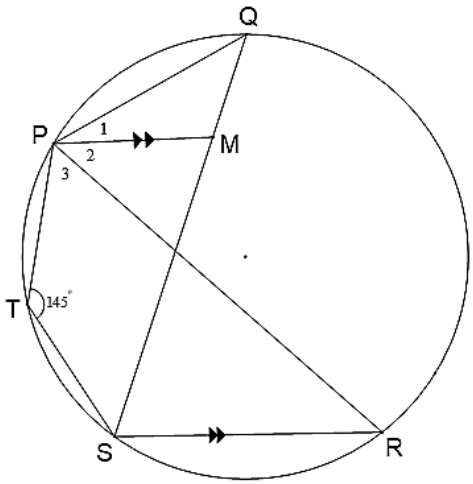
9.12.1	Calculate with reasons, the size of:			
	(a)	$\hat{D}_1$	(1)	**L1**
	(b)	$\hat{B}_1$	(2)	**L1**
	(c)	$\hat{C}_2$	(1)	**L2**
	(d)	$D\hat{A}B$	(2)	**L3**
9.12.2	Is BD a diameter to the circle? Motivate your answer.		(2)	**L2**
			[8]	

9.13



9.13.1	Complete this statement: A line drawn from the centre of a circle is …… to the tangent.		(1)	**L1**
	In the diagram below, AOD is a diameter of the circle and EDF is a tangent to the circle at D. $A\hat{D}B = 62°$ and BC = CD. Calculate, with reasons, the numerical value of:			
9.13.2	$B\hat{C}D$		(4)	**L3**
9.13.3	$C\hat{D}F$		(4)	**L2**
			[8]	

9.14

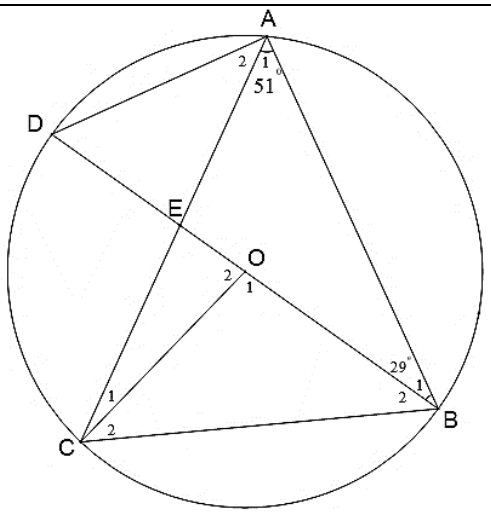


In the figure Q, P, T and R lie on the circle. M lies on QS such that $PM \parallel SR$.

$P\hat{T}S = 145°$.

9.14.1	Calculate the size of:			
	(a)	$\hat{Q}$	(2)	**L1**
	(b)	$\hat{R}$	(2)	**L1**
	(c)	$\hat{P}_2$	(1)	**L1**
9.14.2	Why is PR a tangent to the circle passing through P, M and Q?		(1)	**L2**
			[6]	

9.15



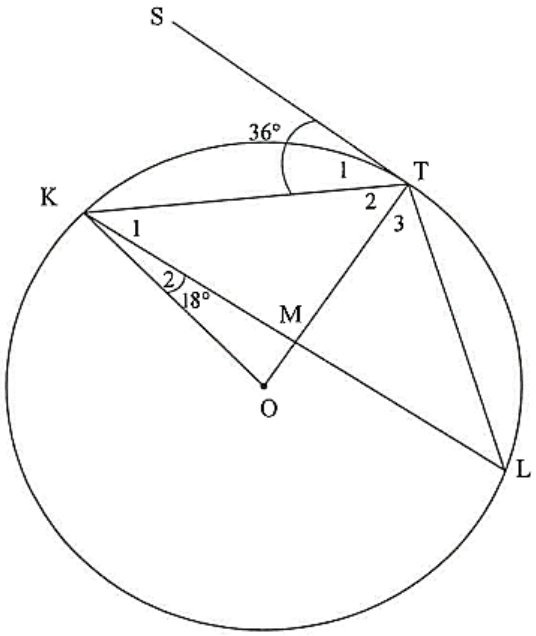
In the diagram, O is the centre of the circle. Points A, B, C and D lie on the circumference of the circle. BOD is a diameter.
AC and BD intersect at E.

$\hat{A}_1 = 51°$ and $\hat{B}_1 = 29°$.

Determine the size of:

9.15.1	$\hat{O}_1$.	(2)	**L1**
9.15.2	$\hat{A}_2$.	(2)	**L2**
9.15.3	$\hat{D}$.	(1)	**L1**
9.15.4	$A\hat{C}O$.	(3)	**L3**
		[8]	

9.16

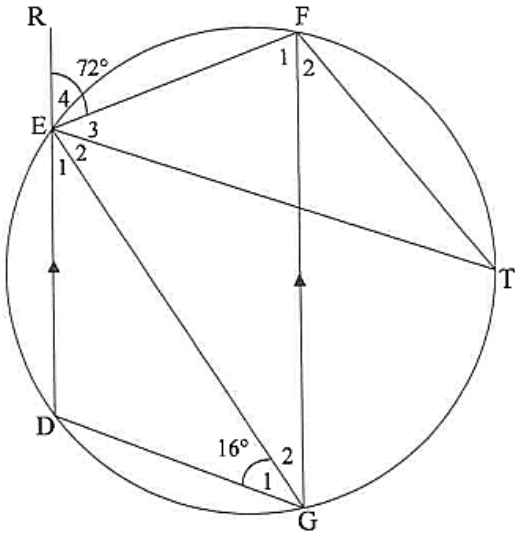


In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersect KL at M. ST is a tangent to the circle at T.

$S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.

9.16.1	Determine, with reasons, the size of:			
	(a)	$\hat{T}_2$	(2)	**L2**
	(b)	$\hat{L}$	(2)	**L1**
	(c)	$K\hat{O}T$	(2)	**L3**
9.16.2	Prove, giving reasons, that $KM = ML$.		(3)	**L3**
			[9]	

9.17

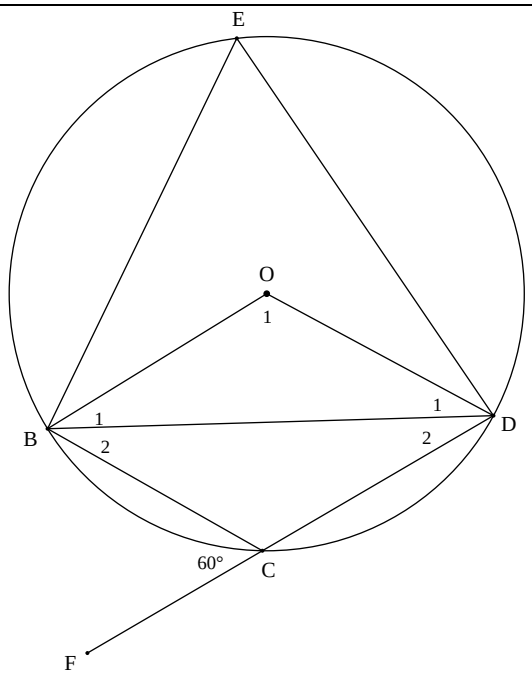


In the diagram, DEFG is a cyclic quadrilateral with $DE \parallel GF$. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.

Determine, with reasons, the size of the following angles:

9.17.1	$D\hat{G}F$	(2)	**L1**
9.17.2	$\hat{T}$	(2)	**L2**
9.17.3	$G\hat{E}F$	(2)	**L2**
		[6]	

9.18

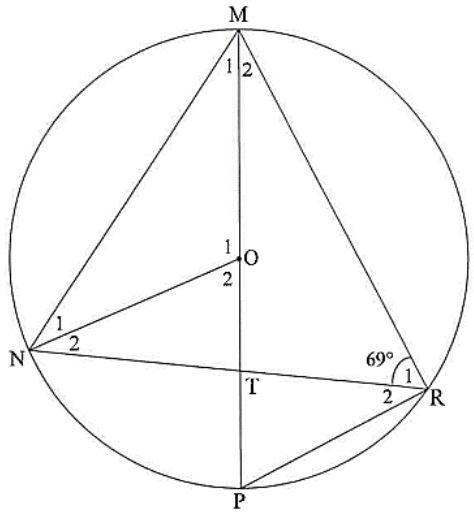


The diagram shows a circle with centre O. BEDC is a cyclic quadrilateral. OB, OD and BD are drawn. DCF is a straight line. $B\hat{C}F = 60°$.

Determine, giving reasons, the size of the following angles:

9.18.1	$\hat{E}$	(2)	**L1**
9.18.2	$\hat{O}_1$	(2)	**L1**
9.18.3	$\hat{D}_1$	(2)	**L2**
		[6]	

9.19

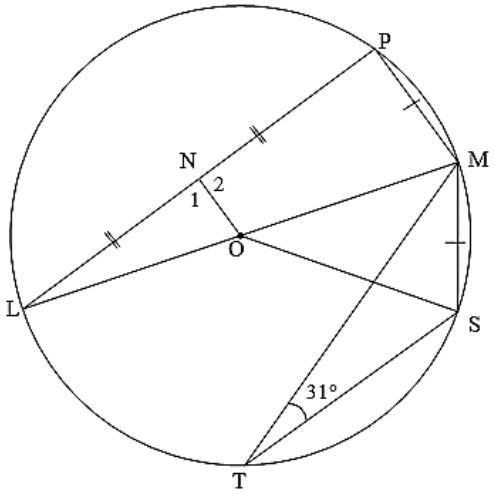


In the diagram, MP is a diameter of a circle centred at O. MP cuts the chord NR at T. Radius NO and chords PR, MN and MR are drawn. $\hat{R}_1 = 69°$.

Determine, with reasons, the size of:

9.19.1	$\hat{R}_2$	(2)	**L2**
9.19.2	$\hat{O}_1$	(2)	**L1**
9.19.3	$\hat{M}_1$	(2)	**L2**
9.19.4	$\hat{M}_2$, if it is further given that $NO \parallel PR$.	(4)	**L3**
		[10]	

9.20

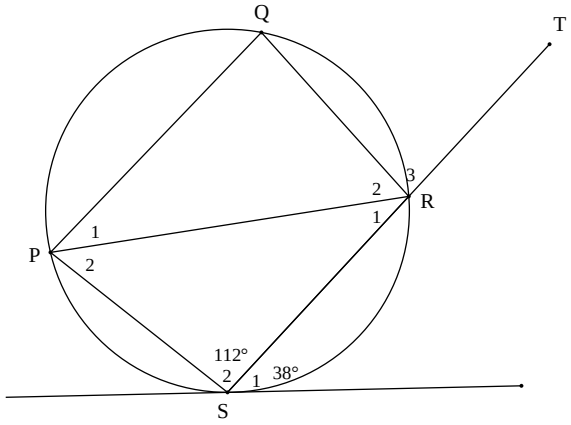


In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn.

$PM = MS$ and $M\hat{T}S = 31°$

9.20.1	Determine, with reasons, the size of each of the following angles:			
	(a)	$M\hat{O}S$	(2)	**L1**
	(b)	$\hat{L}$	(2)	**L1**
9.20.2	Prove that $ON = \frac{1}{2}MS$.		(4)	**L3**
			[8]	

9.21

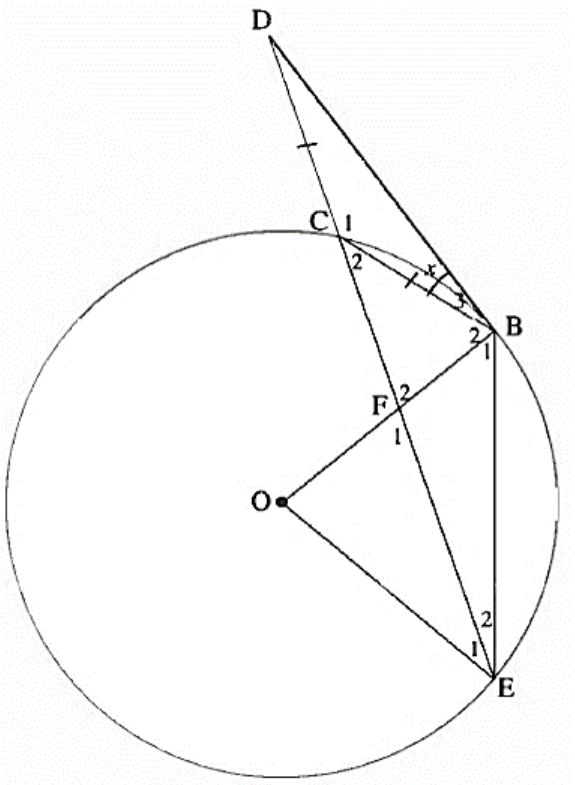


In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. $PQ = QR$, $\hat{S}_1 = 38°$ and $\hat{S}_2 = 112°$.

Determine, with reasons, the size of the following angles:

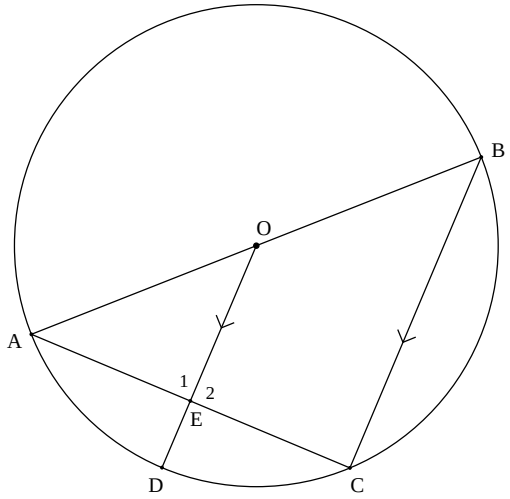
9.21.1	$\hat{Q}$	(2)	**L1**
9.21.2	$\hat{R}_2$	(2)	**L2**
9.21.3	$\hat{P}_2$	(2)	**L1**
9.21.4	$\hat{R}_3$	(2)	**L2**
		[8]	

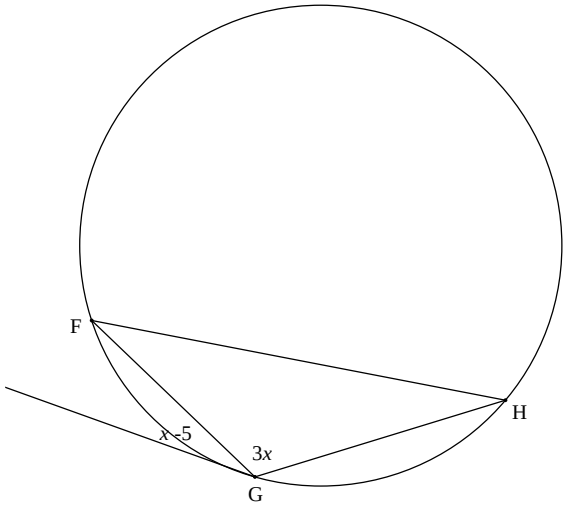
9.22

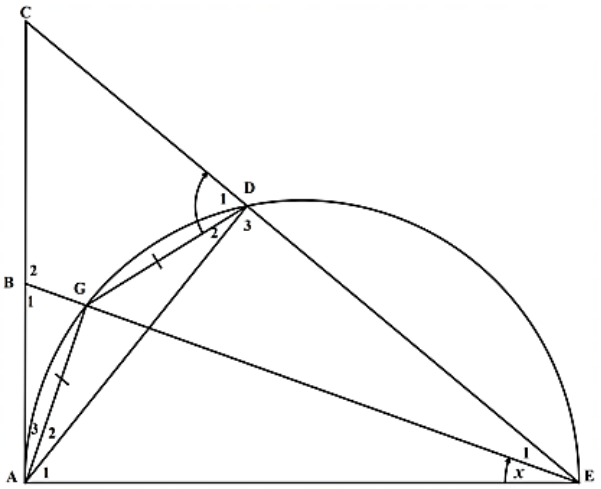


O is the centre of circle CBE. DB is a tangent to the circle at B. EC produced meets BD in D and intersect OB at F. $CD = CB$. OE and BE are joined. $\hat{B}_3 = x$.

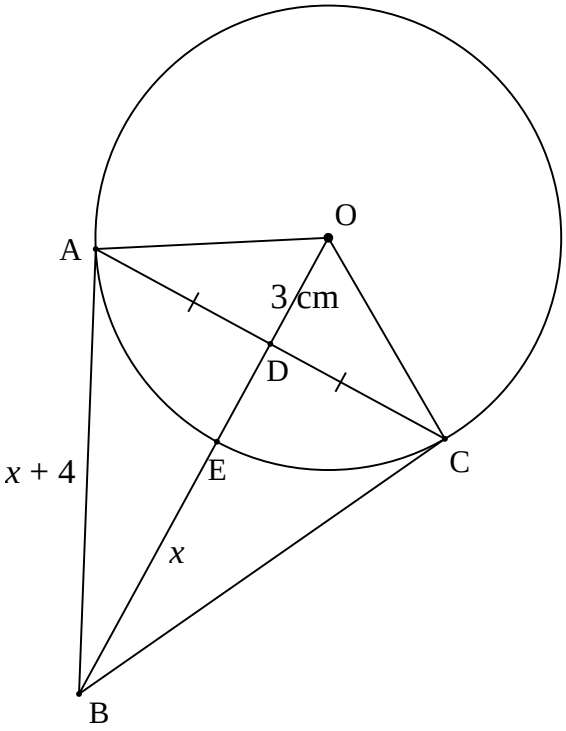
9.22.1	Name, with reasons, two other angles equal to x.	(4)	**L1**
9.22.2	Express $E\hat{O}B$ in terms of x.	(3)	**L3**
9.22.3	Determine, with reasons, the size of $\hat{B}_2$ in terms of x.	(2)	**L3**
9.22.4	Determine, with reasons, the size of $\hat{E}_2$ in terms of x.	(2)	**L3**
		[11]	

<table>
<tr>
<td>9.23</td>
<td>
</td>
<td>O is the centre of the circle. A, B, C and D are points on the circumference. AOB is a straight line. AC and BC are drawn. OD is drawn parallel to BC, and intersects AC at E. The radius of the circle is 10 cm, and $AC = 12$ cm

Calculate the length of ED.</td>
<td>(6)</td>
<td>L3</td>
</tr>
<tr>
<td>9.24</td>
<td>
</td>
<td>In the diagram below, JG is a tangent to the circle FGH at G. FG, GH and FH are drawn. $J\hat{G}F = x - 5$ and $F\hat{G}H = 3x$.

Calculate the value of x.</td>
<td>(5)</td>
<td>L3</td>
</tr>
<tr>
<td>9.25</td>
<td>
</td>
<td>A, G, D and E are points on a semi-circle having AE as the diameter. CA is a tangent to the semi-circle at A. ED produced meets the tangent at C. AG, GD and EG are drawn. EG produced meets the tangent at B. $AG = GD$.
$A\hat{E}B = x$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.25.1 Name, with reasons, THREE other angles equal to x.</td>
<td>(5)</td>
<td>L3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.25.2 Prove that BCDG is a cyclic quadrilateral.</td>
<td>(5)</td>
<td>L4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[10]</td>
<td></td>
</tr>
</table>

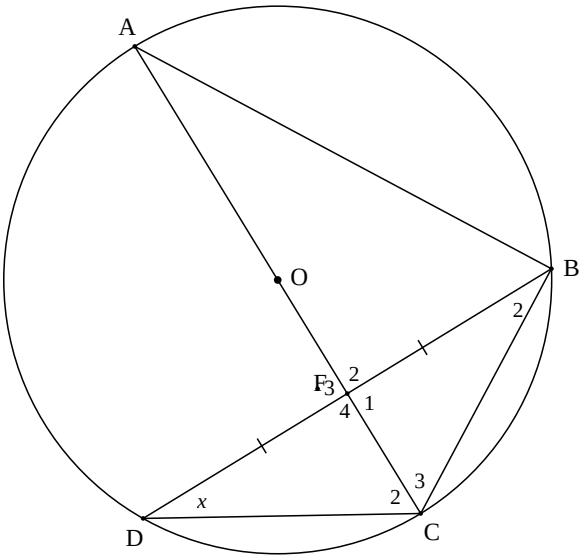
9.26



In the diagram below O is the centre of the circle. Quadrilateral ABCO is drawn with A and C on the circumference. AB is a tangent to the circle at point A and BC is a tangent to the circle at C. D is the midpoint of chord AC and . $AD = DC$. E is the point on the circumference of the circle with centre O.
The length of:
$AC = 8\text{cm}$,
$OD = 3\text{cm}$,
$BE = x$ and
$AB = x + 4$

9.26.1	Write down the size of $\hat{D}_1$ with a reason.	(2)	**L2**
9.26.2	Calculate the length of OA.	(2)	**L1**
9.26.3	Determine the size of $\hat{A}$ and give a reason for your answer.	(2)	**L2**
9.26.4	Calculate the value of x if $x > 1$.	(4)	**L3**
		[10]	

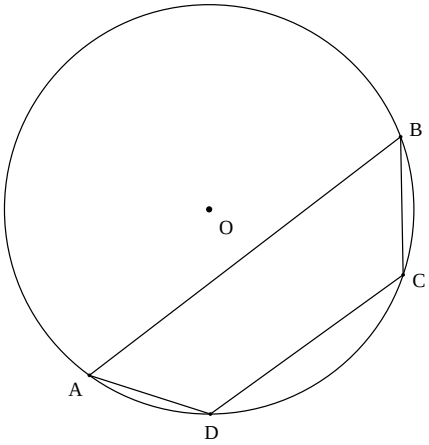
9.27



In the diagram, A, B and D lie on the circle with centre O. AOFC and DFB are straight lines. $DF = FB$, $\hat{D} = x$.

9.27.1	Determine, with reasons. the size of EACH of the the following in terms of x.			
	(a)	$\hat{A}$	(2)	**L1**
	(b)	$\hat{C}_3$	(3)	**L2**
9.27.2	Prove, giving reasons, that $\hat{F}_2 = \hat{F}_3$		(2)	**L2**
			[7]	

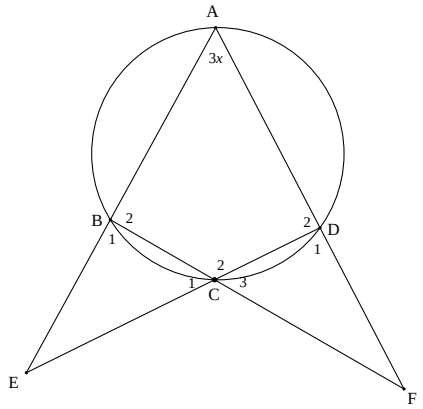
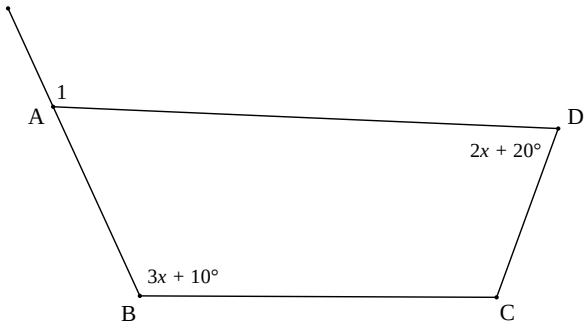
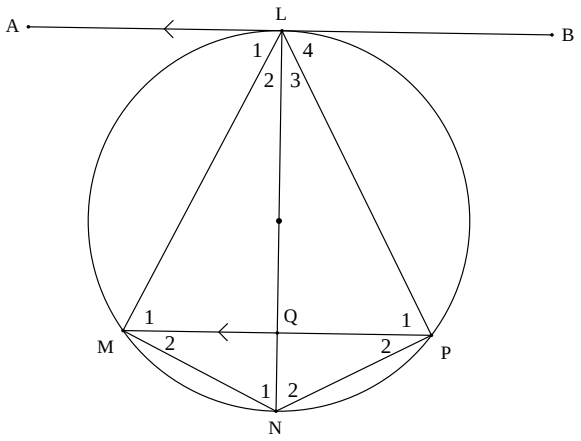
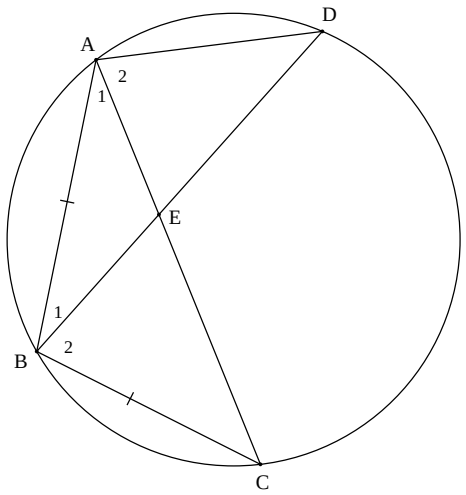
9.28



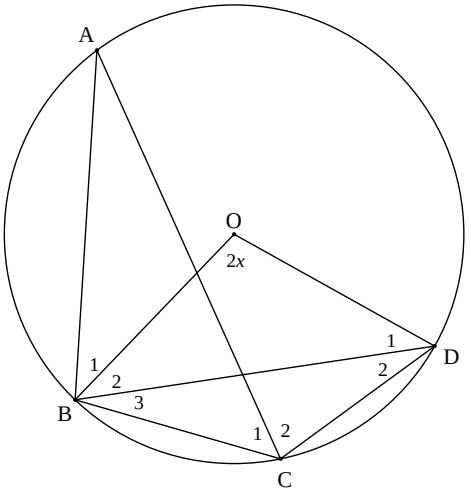
In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.

Use the diagram to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is to prove that $\hat{B} + \hat{D} = 180°$.

(5) **L4**

9.29		In the diagram, ABCD is a cyclic quadrilateral. AB and DC are produced to meet at E. AD and BC are produced to meet at F. $A\hat{F}B = 2x$, $D\hat{A}B = 3x$ and $A\hat{E}D = x$. Determine, giving reasons, the value of x.	(6)	**L4**
9.30		In the diagram, $\hat{A}_1 = \hat{C}$, $\hat{B} = 3x + 10°$ and $\hat{D} = 2x + 20°$. Calculate, with reasons, the value of x.	(5)	**L4**
9.31		ALB is a tangent to circle LMNP. ALB || MP. Prove that:		
		9.31.1 LM = LP	(5)	**L3**
		9.31.2 LN bisects $M\hat{N}P$	(3)	**L2**
		9.31.3 LM is a tangent to circle MNQ.	(3)	**L3**
			[11]	
9.32		In the circle ABCD, AB = BC. Prove that AB is a tangent to the circle AED.	(4)	**L3**

9.33



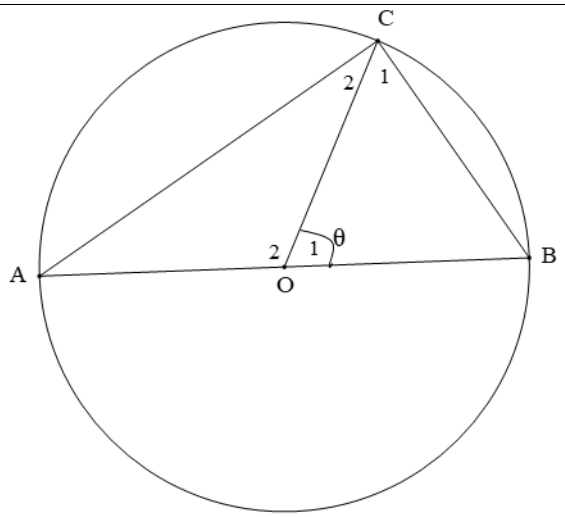
In the diagram, O is the centre of the circle through A, B, C and D.

$BC = CD$ and $B\hat{O}D = 2x$.

Determine the following angles in terms of x.

9.33.1	$\hat{B}_2$	(3)	**L2**
9.33.2	$B\hat{C}D$	(3)	**L3**
9.33.3	$\hat{A}$	(4)	**L4**
		[10]	

9.34

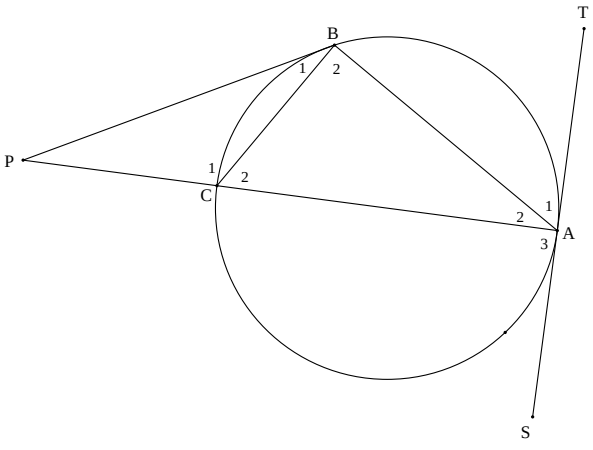


In the figure, AB is a diameter of the circle with centre O and radius r. C is a point on the circle such that $C\hat{O}B = \theta$ (acute angle).

Prove that:

9.34.1	$BC = \dfrac{r\sin\theta}{\cos\frac{\theta}{2}}$	(3)	**L3**

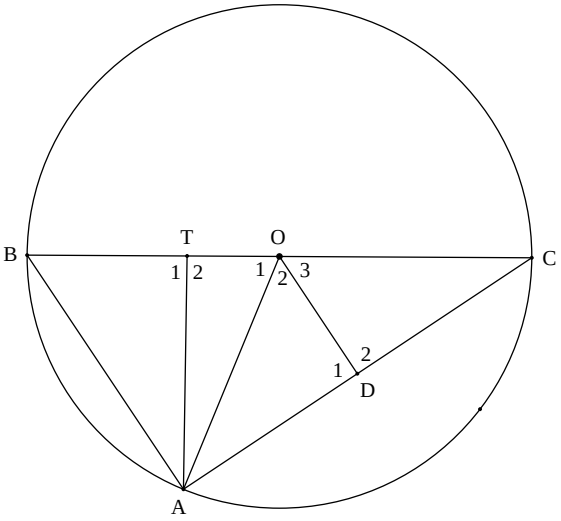
9.35



In the diagram, AC is a diameter of the circle and PB is a tangent. AC produced meets the tangent at P. TAS is also a tangent. $\hat{P} = x$ and $\hat{C}_1 = y$.

Prove that:

9.35.1	$\hat{A}_2 + \hat{A}_3 = y$	(4)	**L3**
9.35.2	$x + 2y = 270°$	(4)	**L4**
		[8]	

	MIND ACTION SERIES			
9.36		BC is a diameter of the circle ABC with centre O. $OD \perp AC$ and $AT \perp BOC$. $BT = x$, $TO = 5$ and $AD = \sqrt{42}$.		
		9.36.1 Prove that $OA = 7$	(5)	**L4**
		9.36.2 Calculate the length of AB.	(4)	**L4**
			[8]	

TOPIC	**10. MEASUREMENT**

Remember that in this section the formulae for calculating the surface area of right prisms are **NOT GIVEN** in the question paper. Hence, one must be able to be able to work out the formulae:

Surface Area

When calculating the surface area of right prisms, first calculate the areas of the polygons making up the figure then add all the individual areas to make up the total surface area.

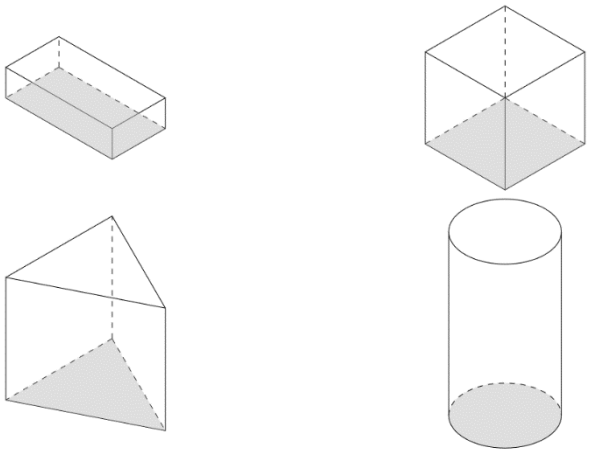
Surface Area = Sum of areas of individual polygons

Volume

When calculating the area of a right prism calculate the area of the base and then multiply by the height

Volume = Area of the base x height

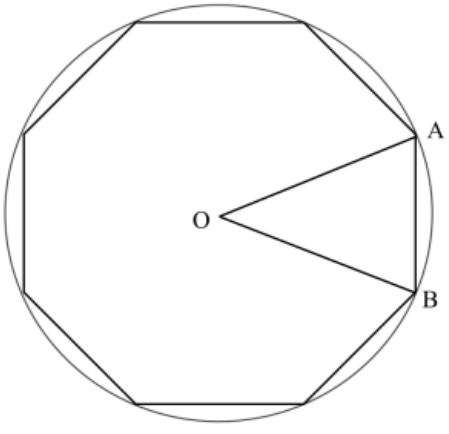
EXAMPLES OF RIGHT PRISMS



The formulae for the volume and surface area of a **cone** and **sphere** are provided in the question.

ACTIVITIES

10.1



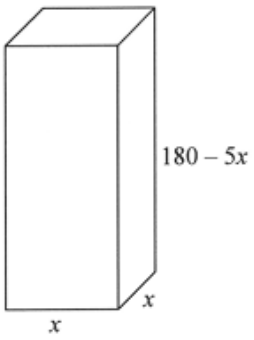
The diagram shows the regular octagon inscribed in a circle of radius r cm and with centre O. A and B are two vertices of the octagon. AO, BO and AB are drawn.

10.1.1 Show that the area of the octagon is $2\sqrt{2}r^2 \text{cm}^2$ (4) **L3**

10.1.2 If $r = 5\text{cm}$, calculate the perimeter of the octagon (3) **L2**

[7]

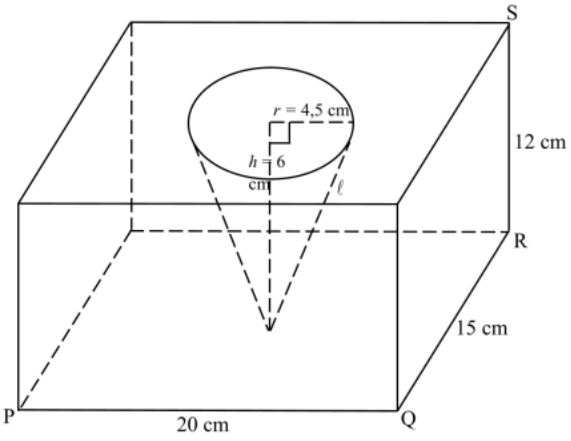
10.2



In the diagram, the right rectangular prism has a square base of x cm and the height of $(180-5x)$ cm.

Calculate the maximum surface area of the prism (5) **L3**

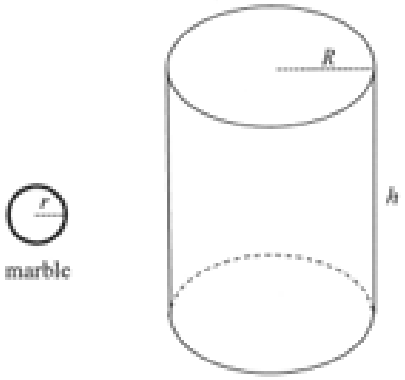
10.3



The solid was made by drilling the right circular cone out of a right rectangular prism (i.e. the cone is removed from the prism)
P, Q, R, and S are the vertices of Prism such that $PQ = 20$ cm, $QR = 15$ cm, $RS = 12$ cm.
The radius of the cone, r , is 4.5 cm and the height of the cone, h, is 6 cm. The slant height of the cone is *l*.

Calculate the total surface area of the solid. (7) **L4**

10.4



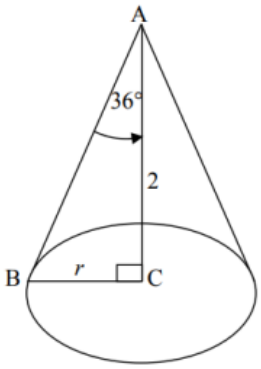
The cylindrical container shown will be filled with spherical marbles. the container has a radius of R cm and a perpendicular height of h cm. The volume of the container is 300 cm^3.
The radius of each marble is 0.75 cm.

10.4.1 Show that the height of the container, h, is given by $h = \dfrac{300}{\pi R^2}$ (2) **L2**

10.4.2 The container is filled with 100 marbles. Then water is poured into the container until the water reaches the top. Calculate the volume of water that was poured into the container. (3) **L4**

[5]

10.5

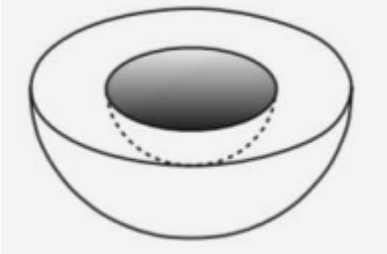


The perpendicular height, AC, of the cone is 2 metres and the radius is r. AB is the slant height.

$B\hat{A}C = 36°$

Calculate the total surface area of the cone. (6) **L3**

10.6



The sketch shows a hemisphere with smaller hemisphere cut out:

the radius of the larger hemisphere is 5 cm and the radius of the smaller hemisphere is 3 cm. Calculate the:

10.6.1 volume (3) **L2**

10.6.2 surface area of the shape. (4) **L3**

[7]